Carbon Emissions and Redistribution: The Design of Carbon Tax Rebates *

Lea Fricke
University of St. Gallen

Clemens Fuest
ifo Institute, University of Munich

Dominik Sachs
University of St. Gallen

August 20, 2025

Abstract

We study optimal redistribution and carbon taxation in a Mirrlees framework. Households differ in their carbon footprint due to both (i) the overall level of spending and (ii) the composition of spending. Introducing a cap on carbon emissions reduces the social value of output, which lowers the efficiency costs of taxation and thereby strengthens the scope for redistribution. However, the optimal increase in redistribution is weaker than suggested by popular proposals for a carbon dividend. While the optimal rebate schedule overcompensates low-income households and undercompensates high-income households for their carbon tax burden, the rebate nevertheless rises with income. Quantifying the model for Germany, we find that the optimal rebate for the 90th income percentile is nearly three times that for the 10th percentile, whereas carbon tax payments are about seven times higher. This results in higher effective average tax rates at the top and lower ones at the bottom of the income distribution.

JEL codes: H21, H23, Q58

Keywords: Carbon Tax, Optimal Taxation

*lea.fricke@student.unisg.ch, fuest@ifo.de, dominik.sachs@unisg.ch. Parts of this research were conducted while Lea Fricke was a visiting researcher at the ifo Institute. We thank Anna Zeitz for excellent research assistance. We thank Karl Schulz, Marcelo Arbex, Xavier Jaravel, Winfried Koeniger, Etienne Lehmann, Arash Nekoi, Lennart Schlattmann and Daniel Weishaar for helpful comments. We further thank seminar and conference participants at IIPF 2024, Swiss Macro Workshop 2025, CY Cergy Paris Université, 2025 Meeting of the Public Finance Committee of the German Economic Association, ZEW Public Finance Conference 2025, Barcelona Workshop on the Economics of Taxation 2025, CESifo Public Economics Week 2025 for comments.

1 Introduction

Carbon pricing has emerged as a key instrument for mitigating climate change. By 2025, 13 countries have implemented a national carbon tax, and 31 countries operate a national emission trading system (ETS). Taken together, these national systems cover about 23% of global greenhouse gas emissions and generate substantial revenues, which have tripled over the last decade (World Bank 2025).[1] The question of how to use carbon pricing revenues has become an important policy issue.

One popular approach, supported by many economists, is to rebate the revenue through lump-sum payments to citizens. This payment is commonly referred to as a carbon dividend. In a public statement, almost 4 000 U.S. economists called for revenue recycling through a carbon dividend:

> *"To maximize the fairness and political viability of a rising carbon tax, all the revenue should be returned directly to U.S. citizens through equal lump-sum rebates. The majority of American families, including the most vulnerable, will benefit financially by receiving more in "carbon dividends" than they pay in increased energy prices. "* (The Wall Street Journal 2019)

This argument emphasizes the distributional effects of a carbon dividend: low-income households receive transfers that exceed their carbon tax burden, effectively financed by high-income households. At first glance, the carbon dividend appears to be a pragmatic policy instrument, valued for its simplicity, transparency, and perceived fairness. Yet it raises a broader question of policy design: should the overall degree of redistribution in the economy increase when climate policies are introduced? And if such an increase is desirable, why has it not already been pursued independently of environmental concerns?

This paper shows that the existence of global warming does indeed have direct and fundamental implications for the optimal degree of income redistribution. Interestingly, these implications are related not just to how rich and poor households are affected by carbon prices but also to a more fundamental issue: the idea that economic growth should slow down or even go into reverse to halt global warming (*degrowth*). Most main-stream economists may not support halting growth to protect the climate, but they do agree that, all else being equal, output that produces carbon emissions is less socially desirable than output that does not. This is true even if innovation reduces the carbon footprint of production or consumption over time but has not yet achieved carbon neutrality.

What does this imply for redistribution policy? In standard models, redistribution is costly because it weakens incentives to work, leading to lower output. In the presence of carbon emissions, however, a lower level of output also reduces emissions – making the welfare loss

[1] In addition to national systems, several countries operate carbon pricing mechanisms at the subnational level. For example, the United States and Japan have implemented ETSs in certain states, while China has introduced carbon taxes in selected provinces. Further, there are supranational emission trading systems such as the EU ETS (World Bank 2025).

from output reductions less severe. The same logic underlying the *degrowth* argument implies that the welfare loss from redistribution becomes smaller — simply because the associated reduction in output is now, to some extent, a desirable side effect.

We formalize this idea by embedding carbon emissions into a version of the standard model of the equity-efficiency trade-off (Mirrlees 1971). We derive comparative statics to examine how a binding carbon constraint reshapes the structure of optimal redistribution. We find that the optimal degree of redistribution increases in the presence of a carbon constraint. The underlying mechanism mirrors the *degrowth* logic: reductions in labor supply become less costly in welfare terms, as they also lower emissions. This makes more redistribution desirable irrespective of the social welfare function. At first glance, this finding seems to support the carbon dividend idea. However, we show that the optimal rebate schedule does not go quite as far as the carbon dividend. Although poorer households are overcompensated and richer households contribute more in net terms, the optimal rebate schedule still increases with income.

We quantify our model using detailed data for the German economy. The optimal rebate for a household at the 90th income percentile is almost three times larger than that for a household at the 10th percentile. However, the associated carbon tax burden at the 90th percentile is roughly seven times higher. Thus, despite the upward-sloping rebate schedule, the policy remains redistributive: the effective average tax rate raises by about one percentage point for households at the 90th percentile and falls by nearly five percentage points at the 10th percentile.

Theory. We consider an economy in which households differ by their skill level and make both labor supply and consumption decisions. Preferences are assumed to be non-homothetic, giving rise to heterogeneous consumption baskets across the income distribution. Carbon emissions result from the consumption of multiple goods, each with different emission intensities. We begin with a status quo scenario in which the government is not concerned with emissions and sets a nonlinear income tax schedule to balance equity and efficiency. We then introduce climate concerns by imposing a carbon cap: the government is now constrained not to exceed a fixed upper bound on aggregate emissions. The primitives of the economy remain unchanged. To meet the emissions target, the government imposes a carbon tax and adjusts the income tax schedule accordingly.[2] The resulting changes in the tax schedule can be interpreted as an income-dependent rebate of the carbon tax revenues. This setup provides a transparent way to isolate the impact of climate constraints on optimal redistribution, allowing for a clean ceteris paribus comparison.

We find that introducing a carbon cap reduces the efficiency costs of redistribution, as labor supply reductions become less costly for the government. Intuitively, a decline in labor

[2] In our setting it is optimal to have a uniform carbon tax. For a recent analysis how this changes if different assumptions on heterogeneity and production technology are made, see Bierbrauer (2025).

supply relaxes the carbon constraint: lower earnings lead to lower consumption and thus to reduced emissions. In standard optimal tax models, the welfare effect of labor supply changes operates through the government's budget constraint – the so-called fiscal externality. In our setting, there is an additional channel: labor supply also affects compliance with the carbon cap, introducing a second constraint through which labor choices impact welfare. This reduction in the marginal efficiency costs shifts the optimal policy toward greater redistribution compared to the status quo. This result is robust. It holds regardless of the specific social welfare criterion and applies both in normative settings with a utilitarian planner and in positive frameworks that use the inverse-optimum approach.

While the carbon cap increases the scope for redistribution, we find that the optimal policy does not go as far as implementing a lump-sum carbon dividend. Instead, the optimal rebate schedule increases with income, reflecting the fact that higher-income households pay more in carbon taxes.

Calibration. We calibrate our model to the German economy in 2018. The main inputs include carbon emission intensities of different goods and services, household consumption expenditure shares, a gross labor income distribution and the current German tax-transfer system. We compute carbon emission intensities for 110 different goods and services, account for both direct and indirect emissions using an environmentally-extended multi-regional supply-use and input-output framework from EXIOBASE. Based on their emission intensities, we aggregate these 110 items into nine broad consumption categories, each comprising a 'brown' and 'green' variety. Household consumption expenditure shares for the these categories are taken from the German Income and Consumption survey (*Einkommens- und Verbrauchsstichprobe*; EVS).

Preferences are parameterized by a two-tier non-homothetic utility function. At the upper-tier, preferences are represented by a non-homothetic CES aggregator over nine goods. At the lower tier, each good is a non-homothetic CES composite of a 'brown' and 'green' variety. This structure allows to capture substitution between goods with differing emission intensities. We calibrate the parameters of the utility function by matching expenditure shares from the EVS along the income distribution, using a minimum-distance estimation approach. Overall, the calibrated utility function provides a good fit to both the consumption allocation patterns and the carbon emission intensity along the income distribution.

Lastly, we calibrate the gross labor income distribution based on German administrative income tax records (*Lohn- und Einkommenssteuerstatistik*; LESt), applying a standard non-parametric kernel density estimation. To capture the German tax-transfer system, we adopt the parametric tax-transfer function of Ferriere, Grübener, Navarro, and Vardishvili (2023), which accommodates the phase-out of transfer receipts. We estimate this function to match the German tax-transfer system as of 2018. Specifically, we use tax return data from the LESt to estimate the parameters of the tax function, and data on transfers from the EVS to estimate the parameters of the transfer function. Our estimated tax-transfer function replicates the sharp drop in marginal tax rates associated with the phase-out of mean-tested transfers.

Quantitative Results. Finally, we evaluate the effect of a carbon cap that achieves a 10% reduction in aggregate emissions on the optimal tax-transfer system. The optimal carbon rebate increases with income: tax units at the 90th percentile receive 2.74 times the rebate of those at the 10th percentile. Hence, in isolation, the carbon rebate is regressive. However, tax units at the 90th percentile pay 7.19 times more in carbon taxes than the 10th percentile. As a result, the carbon policy as a whole is strongly progressive. To further illustrate the progressivity of the carbon policy, we examine changes in effective marginal and average tax rates. The optimal carbon policy raises marginal tax rates above the 25th percentile and increases average tax rates from -40 percentage points at the bottom to 3.25 percentage points at the top. Tax units up to the 64th percentile are net beneficiaries, receiving more in rebates than they pay in carbon taxes.

Comparing the optimal carbon rebate to a carbon dividend policy shows that the carbon dividend is more generous for the bottom 26% of the income distribution and less generous for the top 74%. This implies that a majority would prefer the optimal policy over the carbon dividend. Finally, we perform a battery of robustness checks, using alternative values for externally calibrated parameters. All alternative specifications yield results similar to the benchmark.

Related Literature. The interaction between income and carbon taxation was first been analyzed in the so-called double-dividend literature. In representative agents models, it has been shown that the optimal carbon tax should be below the Pigouvian tax when distortionary income taxes are present, since carbon taxation erodes the income tax base (see, e.g., Bovenberg and De Mooij, 1994; Bovenberg and van der Ploeg, 1994; Goulder, 1995).[3]

Kaplow (2012) and Jacobs and De Mooij (2015) integrate (environmental) externalities into a framework with heterogeneous agents and redistributive income taxation.[4] Both Kaplow (2012) and Jacobs and De Mooij (2015) show that, under weak separability, the optimal carbon tax coincides with the Pigouvian tax, even in the presence of distortionary income taxation.

Kaplow (2012) further shows that for any given introduction of a carbon tax, there exists an income tax reform such that the joint reform is Pareto improving: it is distributionally neutral (in terms of utility) and increases tax revenue. Kaplow (2012, p.488) concludes that : *"[...] concerns about labor supply and distribution [...] are independent of the question of how best to*

[3] A recent modern quantitative treatment of this channel can be found in Barrage (2020) who finds that the optimal carbon tax should be 8-24% lower.

[4] Focussing on a linear income tax, Jacobs and Van der Ploeg (2019) show that if carbon Engel curves are linear, the optimal pollution tax follows a first-best rule. With non-linear carbon Engel curves, however, carbon taxes acquire a redistributive role that may push them above or below the Pigouvian level. Hänsel, Franks, Kalkuhl, and Edenhofer (2022) extend the analyis by adding horizontal inequality in carbon emissions and by consider subsidies for clean energy as an additional policy instrument.

control externalities." In this paper, we take the next step and ask how the additional revenue from carbon taxation should optimally be rebated. We show that irrespective of the welfare function, it should be used in a way that increases redistribution and hence, the concerns on labor supply and distribution are not independent of how externalities are controlled.

Jacobs and de Mooij's (2015) proof that the optimal carbon tax equals the Pigouvian tax builds on the result that the marginal cost of public funds is equal to one (Jacobs 2018). Regarding the rebates of revenue, they argue that it is irrelevant whether the government recycles revenue via lump-sum transfers or income tax reductions. If the tax system is ex-ante optimal, the application of the Envelope Theorem implies that both rebate approaches have the same effect on welfare (Bovenberg and de Mooij, 2015, p. 107). While we agree that the welfare effects of different rebate mechanisms are equivalent at the margin, a comparative static analysis nevertheless reveals the optimal way of recycling the revenue. We explore this path and analyze the implications for redistribution and the design of carbon tax rebate schemes.[5]

More closely related to our research question, Van der Ploeg et al. (2022) take a numerical approach to study how alternative carbon tax rebate schemes affect efficiency, equity, and political feasibility. In a similar vein, Douenne, Hummel, and Pedroni (2025) study optimal linear income taxation and optimal carbon taxes in a dynamic macroeconomic model with inequality. Both papers conclude that the optimal recycling of revenues combines lump-sum transfers with reductions in income taxation.

Our paper also relates to the literature on the implications of environmental problems and natural resource constraints for economic growth. An early and widely discussed contribution to this debate was the report of the Club of Rome (Meadows, Meadows, Randers, and Behrens III 1972), which argued that mankind needed to reign in economic growth in a controlled manner to avoid major natural disasters, which would reduce economic output in a chaotic fashion. Contributions by economists including Nordhaus, Stavins, and Weitzman (1992) criticized this view for neglecting the potential of innovation. More recent contributions to the degrowth debate include Aghion, Boppart, Peters, Schwartzman, and Zilibotti (2025), who argue that economic growth may be sustainable if the economy restructures towards products and services with low carbon footprints.[6]

Structure. We introduce our model in Section 2 and characterize the optimal tax-transfer system in the absence of carbon policies in Section 3. In Section 4 we then turn to the optimal design of carbon rebate policies. In Section 5, we calibrate our model to German economy of 2018. Section 8 concludes.

[5] In recent complementary work, Ahlvik, Liski, and Mäkimattila (2025) consider a Mirrleesian environment with multi-dimensional heterogeneity giving rise to the optimal externality tax being income dependent. They apply their framework to electric vehicles and energy consumption.

[6] Savin and van den Bergh (2024) offer an extensive survey of the recent literature studying economic, social and environmental aspects of degrowth.

2 Model

2.1 Model Basics

Heterogeneity. We consider a static economy with a continuum of heterogeneous households of total mass one, indexed by their productivity $\theta \in [\underline{\theta}, \bar{\theta}]$. Let $H(\theta)$ denote the cumulative distribution function of productivity, with density $h(\theta)$. Households earn a gross labor income given by $y = l\theta$, where l denotes labor supply.

Preferences. Households make a labor-leisure decision and allocate their income across a vector of consumption goods and services $\mathbf{c} = (c_1, c_2, \ldots, c_I)$, purchased at exogenous producer prices $\mathbf{p} = (p_1, p_2, \ldots, p_I)$. Preferences over consumption and labor supply are represented by the utility function

$$U(\mathbf{c}) - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

where $\varphi > 0$ denotes the Frisch elasticity of labor supply. The function $U(\mathbf{c})$ is left unrestricted, allowing for non-homothetic preferences. Importantly, the utility function is weakly separable in consumption and leisure. In the absence of externalities, this implies that uniform commodity taxation is optimal (Atkinson and Stiglitz 1976).

Carbon Emissions. We assume that each unit of consumption of good i generates $f_i > 0$ tons of carbon emissions. The carbon footprint of household θ is given by

$$f(\theta) = \sum_{i=1}^{I} c_i(\theta) f_i$$

where $c_i(\theta)$ denotes the quantity of good i consumed by household θ. The aggregate carbon footprint of the economy is given by

$$F = \int_{\underline{\theta}}^{\bar{\theta}} f(\theta) dH(\theta).$$

Government. The government levies three taxes: a nonlinear income tax schedule $\mathcal{T}(y)$, a uniform ad-valorem commodity tax t_c, and a carbon tax τ_{co_2} per unit of emission. We get back to the government's problem and its budget constraint in Section 2.3.

2.2 Household decision problem given taxes

We divide the household's decision problem into two parts. First, households choose their optimal consumption allocation given net income $e = y - \mathcal{T}(y)$. Second, households choose labor supply given the optimal consumption allocation.

The first stage is given by

$$u(e; \mathbf{p}, t_c, \tau_{co_2}) = \max_{\mathbf{c}} U(\mathbf{c}) \ \text{ s.t. } \sum_{i=1}^{I} c_i \left((1+t_c)p_i + f_i \tau_{co_2}\right) = e. \tag{1}$$

Note that this consumption allocation problem – for a given level of expenditures e – does not depend on productivity θ. The first-order-condition is given by:

$$\forall i: \ \lambda(e; \mathbf{p}, t_c, \tau_{co_2}) = \frac{U_i\left(\mathbf{c}\left(e; \mathbf{p}, t_c, \tau_{co_2}\right)\right)}{\left(1+t_c\right) p_i + f_i \tau_{co_2}}, \tag{2}$$

where $\lambda(e; \mathbf{p}, t_c, \tau_{co_2})$ is the Lagrangian multiplier on the household's budget constraint. By the envelope theorem, it follows that $u_e(e; \mathbf{p}, t_c, \tau_{co_2}) = \lambda(e; \mathbf{p}, t_c, \tau_{co_2})$. Let $c_i(e; \mathbf{p}, t_c, \tau_{co_2})$ denote the optimal consumption of good i, and define the resulting carbon footprint as $f_i(e; \mathbf{p}, t_c, \tau_{co_2})$.

The second stage of the household's problem is given by

$$V(\theta; \mathbf{p}, \mathcal{T}, t_c, \tau_{co_2}) = \max_{y} u(e; \mathbf{p}, t_c, \tau_{co_2}) - \frac{(y/\theta)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \ \text{ s.t. } \ e = y - \mathcal{T}(y). \tag{3}$$

The corresponding first-order condition of the labor supply is given by

$$u_e(e(\theta, \mathbf{p}, \mathcal{T}, t_c, \tau_{co_2}); \mathbf{p}, t_c, \tau_{co_2})\theta \left[1 - \mathcal{T}'(y(\theta, \mathbf{p}, \mathcal{T}, t_c, \tau_{co_2}))\right] = \left(\frac{y(\theta, \mathbf{p}, \mathcal{T}, t_c, \tau_{co_2})}{\theta}\right)^{\frac{1}{\varphi}}. \tag{4}$$

Simplifying Notation. In the following, we suppress the dependence of all variables on the exogenous price vector $\mathbf{p}$ and define the policy vector $\mathcal{P} \equiv (\mathcal{T}, t_c, \tau_{co_2})$. We denote the optimal income and expenditure choices as $y(\theta, \mathcal{P})$ and $e(\theta, \mathcal{P})$, respectively. We often use $u_e(\theta, \mathcal{P})$ as short-hand notation for $u_e(e(\theta, \mathcal{P}); \mathbf{p}, t_c, \tau_{co_2})$, and similarly we write $f(\theta, \mathcal{P}) \equiv f(e(\theta, \mathcal{P}); \mathbf{p}, t_c, \tau_{co_2})$. For ease of notation, we omit dependence on $\mathcal{P}$ when the context is clear.

Comparative Statics. We now define the parameters that govern how households adjust their carbon emissions in response to a change in the carbon tax. First, we define the marginal carbon footprint as the partial effect of net income e on household emissions:

$$f_e(\theta, \mathcal{P}) \equiv \frac{\partial f(\theta, \mathcal{P})}{\partial e}.$$

Second, we define the partial effect of the carbon tax τ_{co_2} on household emissions as

$$f_{\tau_{co_2}}(\theta, \mathcal{P}) \equiv \frac{\partial f(\theta, \mathcal{P})}{\partial \tau_{co_2}}.$$

The total effect of a change in the carbon tax τ_{co_2} on household emissions is then given by (omitting dependence of $\mathcal{P}$):

$$\frac{df(\theta)}{d\tau_{co_2}} = f_{\tau_{co_2}}(\theta) + f_e(\theta)\left(1 - \mathcal{T}'(y(\theta))\right)\frac{\partial y(\theta)}{\partial \tau_{co_2}},$$

where the first part captures the direct effect of the carbon tax on carbon emissions, holding total expenditure constant. The second term captures the indirect effect through the labor supply response to the carbon tax and the resulting change in the level of net income.

We now derive an expression for $\frac{\partial y(\theta)}{\partial \tau_{co_2}}$ that depends on classical labor supply elasticities. This helps to relate the labor supply effects of carbon taxes to the well-known labor supply effects of income taxes. First, the elasticity of income with respect to the retention rate $1 - \mathcal{T}'$ is given by

$$\varepsilon_{y,1-\mathcal{T}'}(\theta) \equiv \frac{d\log(y(\theta))}{d\log(1-\mathcal{T}')} = \frac{1}{\frac{1}{\varphi} + \gamma(\theta)\frac{y(\theta)}{e(\theta)}(1 - \mathcal{T}'(y(\theta))) + \frac{\mathcal{T}''(y(\theta))y(\theta)}{1-\mathcal{T}'(y(\theta))}}, \tag{5}$$

where $\gamma(\theta) = -\frac{u_{ee}(e(\theta))e(\theta)}{u_e(\theta)}$ is the coefficient of relative risk aversion. Second, the income effect parameter is defined as

$$\eta(\theta) \equiv \frac{\partial y(\theta)}{\partial \mathcal{T}(0)} = \frac{\gamma(\theta)\frac{y(\theta)}{e(\theta)}}{\frac{1}{\varphi} + \gamma(\theta)\frac{y(\theta)}{e(\theta)}(1 - \mathcal{T}'(y(\theta))) + \frac{\mathcal{T}''(y(\theta))y(\theta)}{1-\mathcal{T}'(y(\theta))}}. \tag{6}$$

Both, (5) and (6) can be obtained from implicit differentiation of (4).

We now provide a result for the labor supply response to a carbon tax reform $d\tau_{co_2}$:[7]

Lemma 1. *The labor supply change resulting from a marginal increase in the carbon tax $d\tau_{co_2}$ is equivalent to the labor supply change resulting from an income tax reform*

$$d\mathcal{T}(y(\theta)) = f(\theta)d\tau_{co_2}. \tag{7}$$

The change in income is given by:

$$\frac{\partial y(\theta)}{\partial \tau_{co_2}}d\tau_{co_2} = -\frac{y(\theta)}{1-\mathcal{T}'(y(\theta))}\varepsilon_{y,1-\mathcal{T}'}(\theta)f_e(\theta)\left(1-\mathcal{T}'(y(\theta))\right)d\tau_{co_2} + \eta(\theta)f(\theta)d\tau_{co_2}. \tag{8}$$

Proof. See Appendix A.1. □

The first term of (8) captures the substitution effect of the carbon tax on labor supply. An increase in the carbon tax has the same substitution effect as an increase in the marginal income tax rate of[8]

$$d\mathcal{T}'(y(\theta)) = f_e(\theta)\left(1-\mathcal{T}'(y(\theta))\right)d\tau_{co_2}. \tag{9}$$

[7]This is equivalent to Lemma 1 of Saez (2002), who studies the desirability of non-uniform commodity taxes in the presence of preference heterogeneity.

[8]Note that (9) directly follows from differentiating (7).

The result is intuitive as for each additional dollar earned, the government collects $f_e\left(1-\mathcal{T}'\right)$ more through the increase in the carbon tax. This resembles, for example, the impact of a savings tax on labor supply discussed in Ferey, Lockwood, and Taubinsky (2024).

The second term captures the income effect of the carbon tax on labor supply. A marginal increase in the carbon tax reduces household income by $f(\theta)d\tau_{co_2}$, generating an income effect equivalent to that of the income tax reform in (7).

2.3 Government and Policies

Welfare and Constraints. The government maximizes a standard social welfare function,

$$\mathcal{W}(\mathcal{P})=\int_{\underline{\theta}}^{\overline{\theta}}\mathcal{G}\left(V(\theta,\mathcal{P})\right)\omega(\theta)\,dH(\theta), \tag{10}$$

where the weights $\omega(\theta)$ are unrestricted and normalized to integrate to one. We assume that either (i) $\mathcal{G}(x)=x$ or (ii) $\mathcal{G}'(x)>0$, $\mathcal{G}''(x)<0$. The latter captures aversion to inequality of utilities. In the latter case, we further assume that $\mathcal{G}(x)$ is such that coefficient of relative utility-inequality aversion, $-\frac{\mathcal{G}''(V)}{\mathcal{G}'(V)}V$, is non-increasing in V.

We define the social marginal utility of type θ as[9]

$$g(\theta,\mathcal{P})=\frac{\partial\mathcal{W}(\mathcal{P})}{\partial e(\theta)}\frac{1}{h(\theta)}=\mathcal{G}'\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta,\mathcal{P}). \tag{11}$$

The government budget constraint is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}}\left(\mathcal{T}(y(\theta,\mathcal{P}))+\frac{t_c}{1+t_c}e(\theta,\mathcal{P})+\frac{\tau_{co_2}}{1+t_c}f(\theta,\mathcal{P})\right)dH(\theta)\geq\mathcal{R}, \tag{12}$$

where $\mathcal{R}$ is an exogenous revenue requirement. The associated Lagrangian multiplier is denoted by Λ.

Besides the government budget constraint, we assume that the government has to obey a carbon cap constraint

$$\int_{\underline{\theta}}^{\overline{\theta}}f(\theta,\mathcal{P})dH(\theta)\leq\bar{F}, \tag{13}$$

where $\bar{F}$ is an exogenous cap on aggregate carbon emissions. We denote the Lagrangian multiplier of the carbon cap constraint by μ. This constraint captures, for example, limits imposed by international carbon reduction agreements. All our results below would carry over if, instead of a cap, we modeled carbon emissions as an atmospheric externality, with welfare net of the

[9] This definition slightly differs from the standard definition of endogenous social welfare weights going back to Saez (2001), as we do not normalize them by the shadow value of public funds. In our optimal policy formulas below, the reader can interpret the social marginal utility $g(\theta,\mathcal{P})$ as being expressed in units of public funds, since all optimality conditions involve it only through relative terms.

externality given by $\mathcal{W} - D(F)$, where the damage function $D(F)$ is monotonically increasing in aggregate emissions F.

Justification of Uniform Commodity Tax. We focus on a uniform commodity tax because our model builds on the assumption of separable preferences, for which case it is known that the optimal commodity tax is uniform. This extends to the case with a carbon cap as we show below. We could then just assume $t_c = 0$ *w.l.o.g.* since there is one degree of freedom in choosing $\mathcal{T}$ and t_c. We refrain from doing so because it aids in making clear the result about optimal carbon taxes for optimal redistribution. We therefore assume an arbitrary but *exogenous* level of the uniform commodity tax.

Effective Marginal Tax Rate. We define the effective marginal tax rate as the increase in total tax revenue resulting from a one unit increase of household's gross labor income:

$$\tau_{eff}(\theta, \mathcal{P}) = \frac{\mathcal{T}'(y(\theta)) + t_c + \left(1 - \mathcal{T}'(y(\theta))\right) f_e(\theta)\, \tau_{co_2}}{1 + t_c}. \tag{14}$$

Besides the uniform ad-valorem tax t_c and the marginal income tax rate $\mathcal{T}'(\cdot)$, it also accounts for the additional tax revenue generated through higher carbon emissions resulting from a one unit increase in gross household income. The derivation of $\tau_{eff}(\theta)$ is provided in Appendix A.2. Note that $\tau_{eff}(\theta, \mathcal{P})$ is the relevant concept to capture the fiscal externalities that arise from changes in incomes of type θ.

Effective Average Tax Rate. Finally, we briefly define the concept of the effective average tax rate, which we will examine in more detail in the quantitative section. It is given by

$$\bar{\tau}_{eff}(\theta) = \frac{\mathcal{T}(y(\theta)) + \frac{t_c}{1+t_c}\left(y(\theta) - \mathcal{T}(y(\theta))\right) + \frac{\tau_{co_2}}{1+t_c} f(\theta)}{y(\theta)}. \tag{15}$$

2.4 Incidence of Carbon Taxes

We briefly discuss the incidence of a carbon tax. As shown in Lemma 1, the labor supply effect of a carbon tax increase is equivalent to that of an income tax reform defined in (7), implying the same distributional impact. Hence, the two reforms are equivalent in terms of their impact on earnings and redistribution. The key difference between the two tax reforms lies in the carbon tax's additional effect on the allocation of expenditure across goods. This reallocation matters for the incidence since it alters household carbon footprints. Holding income and total expenditures fixed, the carbon footprint of household θ changes by $f_{\tau_{co_2}}(\theta)d\tau_{co_2}$. Hence, a carbon tax hike implies an additional fiscal externality given by

$$\tau_{co_2} \int_{\underline{\theta}}^{\overline{\theta}} f_{\tau_{co_2}}(\theta) d\tau_{co_2} \, dH(\theta). \tag{16}$$

If the carbon cap constraint is not binding, (16) captures the welfare effect of jointly increasing the carbon tax by $d\tau_{co_2}$ and implementing the income tax reform defined in (7), but with the opposite sign. In this case, optimality of the policy $\mathcal{P}$ implies $\tau_{co_2} = 0$. Put differently, when the carbon cap constraint (13) is slack, raising the carbon tax is strictly inferior to implementing the equivalent income tax reform defined in (7).[10] In Section 3, we first consider this benchmark without a binding carbon cap and characterize the optimal 'status quo' policy $\mathcal{P}_{sq} = (\mathcal{T}_{sq}, t_c, 0)$.

In Section 4, we introduce a binding carbon cap, which makes it optimal for the government to impose a positive carbon tax. We characterize the resulting optimal policy as $\mathcal{P}_{co_2} = (\mathcal{T}_{co_2}, t_c, \tau_{co_2})$.[11] We interpret the difference between the two income tax schedules as the optimal carbon tax rebate:

$$R_{co_2}(y) \equiv \mathcal{T}_{sq}(y) - \mathcal{T}_{co_2}(y),$$

and derive its key properties. We also examine whether $\mathcal{P}_{co_2}$ is more or less redistributive than the status quo policy $\mathcal{P}_{sq}$.

3 Normative Benchmark: No Carbon Cap

As a first step, we characterize the optimal tax-transfer system in the absence of a carbon cap constraint, which we refer to as the status-quo policy throughout the following. The government's optimization problem is then given by

$$\max_{\mathcal{T}(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{G}\left(V(\theta; \mathcal{P})\right) \omega(\theta) dH(\theta) \tag{17}$$

subject to a government budget constraint (12) and optimal household behavior (1), (3). Note that the VAT is exogenous and, as shown in Section 2.4, the carbon tax is zero in the absence of a binding cap. Hence, only the first element of the policy vector $\mathcal{P} = (\mathcal{T}, t_c, 0)$ – the income tax schedule – is endogenous.

[10] This logic mirrors the the Atkinson-Stiglitz result on uniform commodity tax: differentiated commodity taxes cannot achieve distributional goals or influence labor supply incentives beyond what income taxes can, but they do introduce additional distortions on the consumption allocation (Atkinson and Stiglitz 1976).

[11] This involves as slight abuse of notation: for $\mathcal{P}$ and $\mathcal{T}$, the subscript $co2$ denotes optimality under the carbon cap constraint, whereas for the carbon tax τ_{co_2}, the subscript was used earlier even when the carbon tax was not set optimally.

The following lemma describes the optimal income tax system in the form presented by Heathcote and Tsujiyama (2021). Its derivation is standard in the literature.[12]

Lemma 2. *Denote the optimal status quo policy by* $\mathcal{P}_{sq} = (\mathcal{T}_{sq}, t_c, 0)$*. The optimal income tax schedule* $\mathcal{T}_{sq}$ *that solves (17) satisfies*

$$\forall\, \theta^* : \; D(\theta^*, \mathcal{P}_{sq}) = E(\theta^*, \mathcal{P}_{sq}),$$

where

$$D(\theta^*, \mathcal{P}_{sq}) = 1 - \frac{E[g(\theta, \mathcal{P}_{sq})|\theta \geq \theta^*]}{E[g(\theta, \mathcal{P}_{sq})]} \tag{18}$$

captures the distributional gains of raising the marginal tax rate $\mathcal{T}'(y(\theta))$ *and*

$$E(\theta^*, \mathcal{P}_{sq}) = 1 - \frac{\frac{1}{1+t_c} - \frac{\tau_{eff}(\theta^*, \mathcal{P}_{sq})}{1-\mathcal{T}'_{sq}(y(\theta^*))}\frac{\varphi}{1+\varphi}\frac{h(\theta^*)\theta^*}{1-H(\theta^*)} + \int_{\theta^*}^{\overline{\theta}} \tau_{eff}(\theta, \mathcal{P}_{sq})\eta(\theta, \mathcal{P}_{sq})\frac{dH(\theta)}{1-H(\theta^*)}}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\overline{\theta}} \tau_{eff}(\theta, \mathcal{P}_{sq})\eta(\theta, \mathcal{P}_{sq})dH(\theta)}$$

captures the efficiency cost of raising the marginal tax rate. The optimal marginal tax rate on income is implicitly defined through the effective marginal tax rate and the uniform VAT:

$$\tau_{eff}(\theta, \mathcal{P}_{sq}) = \frac{\mathcal{T}'_{sq}(y(\theta)) + t_c}{1 + t_c}. \tag{19}$$

Proof. See Appendix A.3. □

The optimal marginal tax rate $\mathcal{T}'_{sq}(y(\theta^*))$ balances optimally the distributional gains $D(\theta^*, \mathcal{P}_{sq})$ with the efficiency cost $E(\theta^*, \mathcal{P}_{sq})$. The term $D(\theta^*, \mathcal{P}_{sq})$ measures the social marginal utility of individuals earning more than $y(\theta^*)$ relative to the population average. Thus, it captures the gains from redistributing incomes from individuals above $y(\theta^*)$ to those below it.

The term $E(\theta^*, \mathcal{P}_{sq})$ captures the efficiency costs arising from changes in labor supply. Since changes in income affect welfare through fiscal externalities, they are weighted by the effective marginal tax rate $\tau_{eff}(\theta, \mathcal{P}_{sq})$, which measures the marginal fiscal effect of earning an additional unit of income. The efficiency costs $E(\theta^*, \mathcal{P}_{sq})$ consists of three such labor supply changes. First, the second term in the numerator reflects the standard substitution effect: a higher marginal tax rate for individuals with income $y(\theta^*)$ reduces their labor supply. Second, the third term in the numerator captures the positive fiscal externalities from income effects, as individuals with income above $y(\theta^*)$ increase their labor supply. Finally, the denominator reflects the negative fiscal externalities due to income effects when tax revenue is redistributed in a lump-sum fashion.

We now briefly discuss the role of t_c. As noted earlier, the values of t_c and $\mathcal{T}'(y(\theta))$ are generally indeterminate: for any given t_c, there exists an income tax schedule $\mathcal{T}$ that implements

[12] A minor difference is that we do not assume $t_c = 0$, but allow for any exogenous value for t_c. As explained above, the value of t_c is generally indeterminate: for any change in t_c, there exists a corresponding adjustment in the income tax schedule that implements the same allocation.

the same allocation. The presence of t_c in the optimality condition highlights that the VAT affects both efficiency costs and distributional gains in the same way as the marginal income tax rate $\mathcal{T}'(y(\theta))$. Ceteris paribus, a higher t_c increases the efficiency cost term and reduces the distributional gains. This equivalence, however, does not extend to carbon taxes, as will be demonstrated in the following section.

4 Optimal Carbon Rebate Policies

We now turn to the case in which the government faces a binding carbon constraint (13). The optimal policy consists of a nonlinear income tax schedule $\mathcal{T}(y)$ and a carbon tax τ_{co_2} that together ensure compliance with the emission cap. The optimal implementation of the carbon cap requires solving the following problem:

$$\max_{\mathcal{T}(\cdot),\tau_{co_2}} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{G}\left(V(\theta;\mathcal{P})\right)\omega(\theta)\,dH(\theta) \tag{20}$$

subject to the carbon cap constraint (13), the government budget constraint (12), and optimal household behavior (1) and (3). As opposed to (17), the government now optimally chooses the first and third element of the policy vector $\mathcal{P} = (\mathcal{T}, t_c, \tau_{co_2})$. The uniform VAT t_c remains exogenous *w.l.o.g.* We denote the solution to this problem by $\mathcal{P}_{co_2} = (\mathcal{T}_{co_2}, t_c, \tau_{co_2})$.

Note that the differences between $\mathcal{T}_{co_2}$ and $\mathcal{T}_{sq}$ arise from the carbon cap constraint and the associated use of the carbon tax. Equivalently, we can fix the income tax schedule at $\mathcal{T}_{sq}$ and introduce a carbon tax rebate function $R_{co2}(y)$ defined by

$$\forall y: \;\; R_{co_2}(y) = \mathcal{T}_{sq}(y) - \mathcal{T}_{co_2}(y).$$

This representation facilitates the analysis of how carbon tax revenues should be optimally rebated across the income distribution.

We present our results sequentially. Section 4.1 analyzes the optimal carbon tax τ_{co_2}. Section Section 4.2 examines the equity-efficiency trade-off. Finally, Section 4.3 explores the properties of the carbon rebate function and assesses whether the optimal policy $\mathcal{P}_{co_2}$ is more redistributive than $\mathcal{P}_{sq}$.

4.1 Optimal carbon tax

Lemma 3. *The carbon tax rate that solves the optimization problem (20) is given by*

$$\tau_{co_2} = \frac{\mu}{\Lambda/(1+t_c)}, \tag{21}$$

where Λ is the Lagrangian multiplier on the government budget constraint (12) and μ is the Lagrangian multiplier on the carbon cap constraint (13).

Proof. See Appendix A.4.1 □

This formula for the carbon tax represents a first-best rule: it holds even if the government could implement type-dependent lump-sum taxes. This result resembles Kaplow (2012) and Jacobs and De Mooij (2015), who show that – under the Atkinson-Stiglitz assumptions – the optimal Pigouvian tax follows a first-best rule, even in a second-best setting with income taxation. [13]

As established in Lemma 1 and further discussed in Section 2.4, a distributionally equivalent income tax reform (7) replicates both the redistributive and labor supply effects of a carbon tax. To derive the condition in Lemma 3, we consider a joint perturbation: a marginal increase in the carbon tax $d\tau_{co_2}$ combined with the distributionally equivalent income tax reform defined in (7), taken with the opposite sign, i.e. $d\mathcal{T}(y(\theta)) = -f(\theta)d\tau_{co2}$. By construction, the reform leaves the distribution of income and labor supply unchanged. The welfare effect therefore arises solely from changes in carbon emissions and the associated carbon tax revenue. These changes in household carbon footprints $f_{\tau_{co2}}(\theta)$ influence welfare only through the government budget constraint and the carbon cap constraint:

$$\left(\Lambda\frac{\tau_{co_2}}{1+t_c} - \mu\right)\int_{\underline{\theta}}^{\overline{\theta}} f_{\tau_{co_2}}(\theta)d\tau_{co_2}dH(\theta). \tag{22}$$

Setting (22) equal to zero immediately yields (21).[14]

4.2 Equity-Efficiency Trade-Off

We now derive the counterpart to Lemma 2. As an intermediate step, we analyze the implications of Lemma 3 for the welfare effects of labor supply responses. A fundamental insight is that the welfare effect now extends beyond the fiscal externality: changes in labor supply also have a first-order welfare impact by tightening or relaxing the carbon cap constraint.

Proposition 1. *The effect of an income change $dy(\theta)$ on social welfare is given by*

$$\Lambda \mathrm{T}(\theta,\mathcal{P}_{co_2})dy(\theta) < \Lambda\tau_{eff}(\theta,\mathcal{P}_{co_2})dy(\theta)$$

where we denote $\mathrm{T}(\theta,\mathcal{P}_{co_2}) \equiv \frac{\mathcal{T}'_{co_2}(y(\theta))+t_c}{1+t_c}$ as the labor wedge.

Proof. The welfare effect of a change in labor supply through the fiscal externality is given by $\Lambda\tau_{eff}(\theta,\mathcal{P}_{co_2})dy(\theta)$. In addition, there is welfare effect through the carbon cap constraint,

[13] As shown by Jacobs and De Mooij (2015), this no longer holds when the utility function is non-separable.

[14] Recall that, in the absence of a binding carbon cap constraint, the welfare effect of this composite reform is given by (16), which in turn implies $\tau_{co_2} = 0$.

given by $-\mu\left(1-\mathcal{T}'_{co_2}(y(\theta))\right) f_e(\theta)dy(\theta)$. Using (21) and (14), these two effects add up to $\Lambda \mathrm{T}(\theta)dy(\theta)$. □

This proposition states that, under a binding a carbon cap, the welfare impact of an increase in labor supply increase is smaller than the corresponding fiscal externality: $\Lambda \mathrm{T}(\theta, \mathcal{P}_{co_2}) < \Lambda \tau_{eff}(\theta, \mathcal{P}_{co_2})$. The reason is that an increase in labor supply not only generates a positive fiscal externality, but also tightens the carbon cap constraint, creating an additional negative welfare effect. Intuitively, an increase in labor supply raises household income, leading to higher consumption, and consequently a larger carbon footprint. Ceteris paribus, this reduces the efficiency costs of redistribution: labor supply reductions become less costly as they relax the carbon cap constraint. This insight is crucial for our result on the equity-efficiency trade-off, as summarized in the following proposition:

Proposition 2. *Consider a government that solves (20). The optimal carbon tax satisfies (21). The optimal income tax schedule* $\mathcal{T}_{co_2}$ *satisfies*

$$\forall\, \theta^* : \; D(\theta^*, \mathcal{P}_{co_2}) = E(\theta^*, \mathcal{P}_{co_2}),$$

where

$$D(\theta^*, \mathcal{P}_{co_2}) = 1 - \frac{E[g(\theta, \mathcal{P}_{co_2})|\theta \geq \theta^*]}{E[g(\theta, \mathcal{P}_{co_2})]} \tag{23}$$

and

$$E(\theta^*, \mathcal{P}_{co_2}) = 1 - \frac{\frac{1}{1+t_c} - \frac{\mathrm{T}(\theta^*, \mathcal{P}_{co_2})}{1-\mathcal{T}'_{co_2}(y(\theta^*))} \frac{\varphi}{1+\varphi} \frac{h(\theta^*)\theta^*}{1-H(\theta^*)} + \int_{\theta^*}^{\overline{\theta}} \mathrm{T}(\theta, \mathcal{P}_{co_2}) \eta(\theta, \mathcal{P}_{co_2}) \frac{dH(\theta)}{1-H(\theta^*)}}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\overline{\theta}} \mathrm{T}(\theta, \mathcal{P}_{co_2}) \eta(\theta, \mathcal{P}_{co_2}) dH(\theta) d\theta}.$$

The optimal marginal income tax rates $\mathcal{T}'_{co_2}(y(\theta)$ *follow implicitly from the labor wedge:*

$$\mathrm{T}(\theta, \mathcal{P}_{co_2}) = \frac{\mathcal{T}'_{co_2}(y(\theta)) + t_c}{1+t_c}. \tag{24}$$

Proof. See Appendix A.4.2. □

The key difference to Lemma 2 lies in the efficiency term $E_{co_2}(\theta^*, \mathcal{P}_{co_2})$, which includes the labor wedge $\mathrm{T}(\theta, \mathcal{P}_{co_2})$ rather than the effective marginal tax rate $\tau_{eff}(\theta, \mathcal{P}_{co_2})$. For $\tau_{co_2} > 0$ and $f_e(\theta) > 0$, we have $\mathrm{T}(\theta, \mathcal{P}_{co_2}) < \tau_{eff}(\theta, \mathcal{P}_{co_2})$, implying that, ceteris paribus, the efficiency costs of redistribution are lower than in standard calculations based on the effective marginal tax rate $\tau_{eff}(\theta, \mathcal{P}_{co_2})$. By contrast, the formula for the welfare gains from redistribution $D_{co_2}(\theta^*, \mathcal{P}_{co_2})$ is not affect by the binding carbon cap.

Hence, Proposition 2 shows that the carbon tax τ_{co_2} asymmetrically affects the equity-efficiency trade-off: for a given allocation, it reduces the efficiency costs of redistribution while leaving the welfare gains from redistribution unchanged. This contrasts with the commodity tax t_c, which symmetrically affects the trade-off by altering both the efficiency costs and the welfare gains from redistribution. It thus highlights that the presence of a carbon tax has nuanced implications for optimal redistribution. Overall, the presence of a binding carbon cap relaxes the equity-efficiency trade-off. Note that this result could easily be overlooked because $\tau_{eff}(\theta, \mathcal{P}_{sq}) = \mathrm{T}(\theta, \mathcal{P}_{co_2})$. Substituting $\tau_{eff}(\theta, \mathcal{P}_{sq})$ into Lemma 2 and $\mathrm{T}(\theta, \mathcal{P}_{co_2})$ into Proposition 2 could mistakenly suggest that optimal redistribution and optimal carbon pricing are tangential.[15]

In the next subsection, we present comparative statics results, explicitly addressing how $\mathcal{T}_{co_2}$ differs from $\mathcal{T}_{sq}$. This analysis (i) compares the level of redistribution with and without the carbon taxes, and (ii) derives key properties of the optimal carbon rebate function R_{co_2}.

4.3 Comparative Statics

We now provide comparative statics for $\mathcal{P}$ to address two questions:

1. Is the optimal policy in the presence of a binding carbon cap $\mathcal{P}_{co_2}$ more or less redistributive than the status quo policy $\mathcal{P}_{sq}$?

2. What are the properties of the carbon tax rebate function $R_{co_2}(y) = \mathcal{T}_{sq}(y) - \mathcal{T}_{co_2}(y)$?

While the two questions are distinct, they are closely linked: both require comparing the optimal marginal tax rates $\mathcal{T}'_{sq}(y(\theta))$ and $\mathcal{T}'_{co_2}(y(\theta))$, as characterized by Lemma 2 and Proposition 2. This comparison is subtle, because even the common terms differ in value due to their evaluation under distinct policies $\mathcal{P}_{sq} \neq \mathcal{P}_{co_2}$.

To obtain tractable comparative statics, we impose a set of simplifying assumptions. These assumptions apply only to the theoretical results in this section and are *not* imposed in the quantitative analysis in Section 5.

Assumption 1. *The carbon emission cap $\bar{F}$ is marginally binding: $\bar{F} = F^{sq} + \varepsilon$, where $\varepsilon \to 0$ and F^{sq} denotes the aggregate carbon footprint under the status quo policy $\mathcal{P}_{sq}$.*

This assumption allows us to focus on the first-order effects of carbon taxes on individual utility and behavior as well as government revenue.

Assumption 2. *The utility function $U(\boldsymbol{c})$ is homothetic and quasi-linear:*

$$U(\boldsymbol{c}) = \left(\sum_{i=1}^{I} \Omega_i^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

[15] Proposition 4 in Jacobs and De Mooij (2015) and Theorem 1 in Ahlvik, Liski, and Mäkimattila (2025) are the counterparts of these papers to our Proposition 2.

This specification implies linearity of $u(e;\mathcal{P})$ in e and hence zero income effects $\eta(\theta;\mathcal{P}) = 0\ \forall\theta$ and linear carbon Engel curves: $f(\theta)/e(\theta) = f(\theta')/e(\theta') = \alpha\ \forall\theta,\theta'$, which in turn implies $f_e(\theta) = \alpha\ \forall\theta$.

This assumption serves two purposes. First, $\eta(\theta;\mathcal{P}_{co2}) = \eta(\theta;\mathcal{P}_{sq}) = 0$ greatly simplifies the comparative statics analysis for the efficiency cost terms, which reduce to:

$$E(\theta^*,\mathcal{P}_{co_2}) = \frac{\mathrm{T}(\theta^*,\mathcal{P}_{co_2})}{1-\mathcal{T}'_{sq}(y(\theta^*))}\frac{\varphi}{1+\varphi}\frac{h(\theta^*)\theta^*}{1-H(\theta^*)} \tag{25}$$

and

$$E(\theta^*,\mathcal{P}_{sq}) = \frac{\tau_{eff}(\theta^*,\mathcal{P}_{sq})}{1-\mathcal{T}'_{co_2}(y(\theta^*))}\frac{\varphi}{1+\varphi}\frac{h(\theta^*)\theta^*}{1-H(\theta^*)}. \tag{26}$$

Second, linear Engel curves imply that a carbon tax does not generate heterogeneous inflation rates in gross prices. This simplifies comparative statics for the distributional gains term $D(\theta,\mathcal{P})$, as will shown below.[16]

Assumption 3. *The primitives of the economy and the welfare function are such that the optimal status quo income tax schedule $\mathcal{T}_{sq}$ is affine, i.e. it features a constant marginal tax rate: $\mathcal{T}'_{sq}(y) = \mathcal{T}'_{sq}(y')$ for all y, y'.*

This assumption does *not* restrict the structure of $\mathcal{T}'_{co_2}$; it merely implies that the social planner optimally chooses an affine income tax schedule in the absence of a carbon cap. It is helpful for deriving sharper results, as it ensures that a household's marginal income tax rate $\mathcal{T}'_{sq}(y)$ is unaffected by small income changes induced by the carbon tax and the corresponding adjustments in the lump-sum rebate.

4.3.1 Result 1: A Carbon Cap Increases the Optimal Level of Redistribution

We now show that the optimal level of redistribution is higher in the presence of the carbon cap:

$$\tau_{eff}(\theta,\mathcal{P}_{co_2}) \geq \tau_{eff}(\theta,\mathcal{P}_{sq})\ \forall\theta.$$

To establish this, we define a distributionally neutral carbon rebate as a benchmark:

$$\forall\,\theta:\ R_{co_2}(y(\theta,\mathcal{P}_{co_2})) = \tau_{co_2} f(\theta,\mathcal{P}_{co_2}). \tag{27}$$

Such an implementation of the carbon cap implies a rebate schedule that exactly offsets each household's carbon tax burden. Under this policy, the level of redistribution is the same as under the status quo, and it holds that $\tau_{eff}(\theta,\mathcal{P}_{co2}) = \tau_{eff}(\theta,\mathcal{P}_{sq})\ \forall\theta$. We now demonstrate

[16] As shown by Jaravel and Olivi (2024), heterogeneous inflation rates have subtle implications for optimal redistributive policy because the marginal value of one dollar, measured in terms of consumption baskets, changes differently across the income distribution. By assuming linear Engel curves, we abstract from this channel. In the quantitative section below, the channel is present.

that the optimality condition in Proposition 2 would be systematically violated, in the sense that $E(\theta, \mathcal{P}_{co_2}) < D(\theta, \mathcal{P}_{co2})$.

Proposition 3. *Assume that Assumptions 1-3 hold. The optimal policy* $\mathcal{P}_{co_2} = (\mathcal{T}_{co_2}, t_c, \tau_{co_2})$ *cannot satisfy (27), because this would imply* $E(\theta, \mathcal{P}_{co_2}) < D(\theta, \mathcal{P}_{co_2})\ \forall\ \theta\ \in]\underline{\theta}, \overline{\theta}[$ *and hence the government would have an incentive to increase redistribution, i.e. decrease* $R'_{co_2}(y(\theta))$ *(increase* $\mathcal{T}'_{co_2}(y(\theta))$ *respectively).*

Proof. See Appendix A.4.3. □

We first consider the distributional gains term and examine how the social welfare weights $g(\theta, \mathcal{P})$ change. Since the distributionally neutral policy leaves the levels of utility unchanged, only changes in the marginal utility of expenditure $u_e(\theta)$ can affect the distributional gains term. As shown in Appendix A.4.3,

$$\forall \theta : u_e(\theta, \mathcal{P}_{co_2}) = u_e(\theta, \mathcal{P}_{sq})\,(1 - \tau_{co_2}\alpha)\,. \tag{28}$$

Equation (28) implies that the relative change in marginal utilities is identical across households, leaving the distributional gains term unaffected: $D(\theta, \mathcal{P}_{co_2}) = D(\theta, \mathcal{P}_{sq})\ \forall\ \theta$, where $D(\theta, \mathcal{P}_{co_2})$ denotes the distributional gains from marginally increasing $\mathcal{T}'_{co_2}(y(\theta))$ starting from the distributional neutral policy. Put differently, starting from a distributionally neutral carbon rebate (27), the distributional gains are exactly the same as before the introduction of the carbon policy.

Next, we turn to the efficiency costs of redistribution. Recall that under Assumption 2 $E(\theta, \mathcal{P}_{co_2})$ and $E(\theta, \mathcal{P}_{sq})$ simplify to (25) and (26). Note that the distributionally neutral policy (27) implies $\forall\ \theta\ \tau_{eff}(\theta, \mathcal{P}_{sq}) = \tau_{eff}(\theta, \mathcal{P}_{co_2})$, i.e.

$$\mathcal{T}'_{co_2}(y(\theta)) = \mathcal{T}'_{sq}(y(\theta)) - \left(1 - \mathcal{T}'_{sq}(y(\theta))\right) f_e(\theta, \mathcal{P}_{co_2})\tau_{co_2}. \tag{29}$$

Substituting (29) into $\mathrm{T}(\theta, \mathcal{P}_{co_2})$ and (25) then yields:

$$E(\theta, \mathcal{P}_{co_2}) < E(\theta, \mathcal{P}_{sq})\ \forall\ \theta\ \in]\underline{\theta}, \overline{\theta}[,$$

which implies that $E(\theta, \mathcal{P}_{co_2}) < D(\theta, \mathcal{P}_{co_2})\ \forall\ \theta\ \in]\underline{\theta}, \overline{\theta}[$. Hence, except at the boundaries $\theta^* = \underline{\theta}$ and $\theta^* = \overline{\theta}$, it is optimal to raise marginal tax rates from (29) implying $\tau_{eff}(\theta, \mathcal{P}_{co_2}) > \tau_{eff}(\theta, \mathcal{P}_{sq})$. The optimal level of redistribution therefore increases, regardless of the form of the social welfare function. The rationale is akin to the arguments underlying the *degrowth* idea. Carbon externalities make, ceteris paribus, reductions in labor supply and consumption more desirable, as these directly lower the carbon footprint.

4.3.2 Result 2: The Optimal Policy is Less Progressive than the Carbon Dividend

We now demonstrate that a carbon dividend cannot be optimal. To this end, we assume that the government rebates the revenue from the small carbon tax in a lump-sum fashion and show that this violates the optimality condition in Proposition 2.

Proposition 4. *Assume that Assumptions 1-3 hold. The optimal policy $\mathcal{P}_{co_2} = (\mathcal{T}_{co_2}, t_c, \tau_{co_2})$ cannot be consistent with a lump-sum rebate. Such a rebate would imply $E(\theta, \mathcal{P}_{co_2}) > D(\theta, \mathcal{P}_{co_2}) \; \forall \; \theta \in]\underline{\theta}, \overline{\theta}[$ and hence, starting from a budget neutral carbon dividend, the government would have an incentive to reduce redistribution, i.e. increase $R'_{co_2}(y(\theta))$ (decrease $\mathcal{T}'_{co_2}(y(\theta))$ respectively).*

Proof. See Appendix A.4.4. □

First, we examine the change in the distributional gains. Changes in marginal utility are again given by (28), as in the case of a distributionally neutral rebate. Under quasi-linearity, marginal utilities u_e are affected only by the relative price change, not by changes in consumption levels. Thus, u_e is again described by (28), and the ratio of marginal utilities remains unchanged. Yet, utility levels change, and the distribution of utility levels becomes more equal. As show in Appendix A.4.4, this implies:

$$\frac{d\mathcal{G}'}{d\theta} > 0,$$

and therefore $D(\theta, \mathcal{P}_{co_2}) < D(\theta, \mathcal{P}_{sq})$. Intuitively, since the carbon dividend policy increases redistribution relative to the status quo, the welfare gains from additional redistribution are lower than under the status quo.

We now turn to the efficiency costs of redistribution. Since the carbon dividend policy implies $\mathcal{T}'_{co_2}(y(\theta)) = \mathcal{T}'_{sq}(y(\theta))$, (25) and (26) directly imply $E(\theta, \mathcal{P}_{co_2}) = E(\theta, \mathcal{P}_{sq})$. At the margin, the efficiency costs of increasing marginal tax rates are identical to those in the status quo. Since we have shown above that the distributional gains term is lower under the carbon dividend, it follows that

$$E(\theta, \mathcal{P}_{co_2}) > D(\theta, \mathcal{P}_{co_2}) \; \forall \; \theta \in]\underline{\theta}, \overline{\theta}[.$$

Hence, except at the boundaries $\theta = \underline{\theta}$ and $\theta = \overline{\theta}$, the government would have an incentive to reduce redistribution below the level implied by the carbon dividend.

Finally, we consider a knife-edge case in which a carbon dividend is the optimal policy. This occurs when the gains from redistribution between any two types θ' and θ'' are independent of the overall level of redistribution. Under Assumptions 1–3, this holds under a weighted Utilitarian welfare function, i.e. $\mathcal{G}(V) = V$. In this case, the welfare weights $g(\theta, \mathcal{P})$ are independent of $\mathcal{P}$ due to the linearity of $\mathcal{G}$ and linearity of $u(e; \mathcal{P})$ in e.

Corollary 1. *Assume that Assumptions 1-3 hold and that $\mathcal{G}(V) = V$. Then, the optimal rebate is lump-sum; that is, a carbon dividend is optimal.*

5 Quantitative Model

We calibrate our model to the German economy in 2018 to quantify the optimal carbon policy. This requires specifying both the set of goods considered and the functional form of utility derived from these goods, which we introduce in Section 5.1. The main inputs to the calibration/model are (i) carbon emission intensities of different goods and services, (ii) household consumption expenditure shares, (iii) a gross labor income distribution, and (iv) the current German tax-transfer system. We describe the different datasets that we use in Section 5.2. We describe our calibration and present the model-data fit in Section 5.3.

5.1 Preferences

For the functional form of $U(\mathbf{c})$, we adopt a nested utility structure with non-homothetic CES preferences over the consumption basket $\mathbf{c} = (c_1, c_2, ..., c_I)$. At the upper-tier, preferences are represented by a non-homothetic CES aggregator over $I = 9$ broad consumption categories. At the lower tier, each good $i = 1, \dots I$ is a CES composite of a 'brown' and 'green' variety, capturing substitution possibilities between goods with different carbon emission intensities.

Upper-Tier Utility Function. We consider non-homothetic CES preferences defined over a consumption basket $\mathbf{c} = (c_1, c_2, ..., c_I)$. Following Comin, Lashkari, and Mestieri (2021), the functional form of $U(\mathbf{c})$ is given by[17]

$$U(\mathbf{c}) = \frac{\mathbb{C}(\mathbf{c})^{1-\gamma}}{1-\gamma},$$

where $\gamma \geq 0$ denotes the coefficient of relative risk aversion and $\mathbb{C}$ is a consumption aggregator implicitly defined by

$$\sum_{i=1}^{I} (\Omega_i \mathbb{C}(\mathbf{c})^{\varepsilon_i})^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} = 1$$

with $\sigma > 0$, $\Omega_i > 0 \ \forall i$. We consider the case $\sigma < 1$, where these broader consumption categories are complements. In this case, $\varepsilon_i > 0 \ \forall i$. With non-homothetic preferences, the elasticity of substitution σ is constant across consumption goods, but the expenditure shares households allocate to different goods vary with their net income $e(\theta)$.

The elasticity of consumption of good i w.r.t. to overall expenditure e is given by

$$\xi_i \equiv \frac{\partial \log c_i}{\partial \log e} = \sigma + (1-\sigma)\frac{\varepsilon_i}{\bar{\varepsilon}},$$

[17] To be precise, we follow the notation of an older working paper version of this paper (Comin, Lashkari, and Mestieri 2017).

where $\bar{\varepsilon} = \sum_i = \psi_i \varepsilon_i$ and ψ_i is the expenditure share of good i. Note that goods with $\varepsilon_i > \bar{\varepsilon}$ are luxuries, while those with $\varepsilon_i < \bar{\varepsilon}$ are necessities.

Lower-Tier Utility Function. Each good c_i is a CES composite of a 'brown' and 'green' variety, which differ in their carbon emission intensities. We denote these varieties by c_{ij}, with $j \in \{b, g\}$ indicating the 'brown' (b) and 'green' (g) variety, respectively. Formally,

$$c_i = \left(\sum_{j \in \{b,g\}} \Omega_{ij} c_{ij}^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}},$$

where σ_i denotes the elasticity of substitution between two varieties and $\Omega_{ij} > 0$. This preference structure allows for substitution from environmentally harmful to environmentally friendly goods within a consumption category i when the relative price of brown varieties increases due to the introduction of a carbon tax.

5.2 Data

We base our calibration on three primary data sources: (i) German income and consumption expenditure survey (*Einkommens- und Verbrauchsstichprobe*; EVS), (ii) environmentally-extended multi-regional supply-use and input-output tables from EXIOBASE, and (iii) German administrative tax records (*Lohn- und Einkommenssteuerstatistik*; LESt). We briefly describe each dataset below; further details are provided in Appendix B.1.

EVS. The *Einkommens- und Verbrauchsstichprobe* (EVS) is a representative cross-sectional household survey that provides detailed data on consumption, income and socioeconomic characteristics. We use the 2018 wave to measure household spending across different goods and services along the income distribution. The survey reports consumption expenditures across 110 categories, classified according to the COICOP system. We aggregate these goods and services into nine broad product categories: durables, electricity, food, heat, housing, services, transport, vacation, and other. In addition, we use detailed information on transfer payments to calibrate a transfer function with a gradual phase-out. Finally, we convert the data from the household level to the tax unit level, resulting in a sample of around 26 000 tax units.

EXIOBASE. We use the environmentally-extended multi-regional supply-use and input-output tables from EXIOBASE to compute emission intensities f_{ij} for the 'brown' and 'green' version of each of the nine goods. EXIOBASE links economic activity to environmental data, including information on greenhouse gas/carbon emissions, through a detailed global input-output framework. The tables are reported by industry using the ISIC classification. To align this with the COICOP-classified consumption data in the EVS, we construct a crosswalk from ISIC sectors to COICOP categories. This allows us to estimate the carbon footprint of each good, accounting for both direct from consumption and indirect emissions throughout the production chain.

LESt. The *Lohn- und Einkommenssteuerstatistik* (LESt) is a 10% cross-sectional random sample of German taxpayers based on administrative income tax records. The unit of observation is the tax unit, defined as either an individual including single persons and married couples filing separately, or a married couple filing jointly. We use data from the 2018 wave on annual gross labor income to calibrate a gross income distribution. In addition, we use total income tax payments to calibrate the status-quo tax system. Our sample includes around 2.78 million tax units.

5.3 Calibration

The calibration of our model consists of four steps. First, we compute emission intensities f_i for the 'brown' and 'green' variety of each of the nine goods. Second, we calibrate the parameters of the utility function by targeting expenditures along the income distribution. Third, we calibrate the status quo tax-transfer system $\mathcal{T}(y)$. Lastly, we calibrate a gross income distribution and then infer the skill distribution $h(\theta)$ from the gross labor income distribution and the status-quo tax-transfer system by inverting the labor supply first-order condition.

Throughout the analysis, we set $I = 9$, where each good i corresponds to one of nine broad consumption categories: durables, electricity, food, heat, housing, services, transport, vacation, and other.

5.3.1 Emission Intensities

We compute carbon emission intensities for 110 different goods and services, accounting for both direct and indirect emissions. Direct emissions arise throughout the consumption of products (e.g. driving a gasoline-powered car), while indirect emissions occur throughout the production chain (e.g production of a car). Using an environmentally extended input-output framework, we calculate the carbon footprint associated with one euro of consumption expenditure as

$$\mathbf{f} = \underbrace{\mathbf{E} \cdot \mathbf{L} \cdot \mathbf{M}}_{\equiv \mathbf{f_{ind}}} + \mathbf{f_{dir}},$$

where $\mathbf{f} \in \mathbb{R}^{n \times 1}$ is a vector of carbon footprints per euro of expenditure for each of the $n = 110$ different goods and services. The first term, $\mathbf{f_{ind}}$, captures the carbon footprint from indirect emissions, derived using the environmentally-extended multi-regional supply-use and input-output framework from EXIOBASE. The second term, $\mathbf{f_{dir}}$, captures the carbon footprint from direct emissions.

The vector of indirect carbon emission intensities, $\mathbf{f_{ind}}$, is computed using the input-output framework from EXIOBASE. Since the classification system and structural organization of EXIOBASE and EVS are not directly aligned, we construct a mapping between the two datasets. Specifically, we define a mapping matrix $\mathbf{M}$ that allocates one euro of expenditure in each COICOP category to the sectoral structure used in EXIOBASE. The vector $\mathbf{f_{ind}}$ is then obtain by multiplying the multi-regional Leontief inverse matrix $\mathbf{L}$ with a row vector of emission intensities $\mathbf{E}$ and the mapping matrix $\mathbf{M}$. The Leontief inverse matrix captures the inter-industry relationships and the region of origin of intermediate goods, while $\mathbf{E}$ contains the amount of carbon dioxide emitted per monetary unit of economic output for each intermediary product and region.

Direct emission intensities for each product are obtained by allocating total total direct emissions from EXIOBASE across final consumption goods. Since EXIOBASE reports only aggregate direct emissions, we complement this information with a more detailed breakdown of household-level direct emissions from German Statistical Office (2023), following the methodology of Steen-Olsen, Wood, and Hertwich (2016) and Hardadi, Buchholz, and Pauliuk (2021). The resulting total emission intensities for each product are listed in Table 5 in Appendix B.1. See Appendix B.2 for methodological details.

Each product is then classified as either 'brown' or 'green' within its product category i, based on its carbon footprint. A product is classified as 'brown' if its carbon footprint exceeds the expenditure-weighted median carbon footprint within that category; otherwise, it is classified as 'green'.[18] We then compute the expenditure-weighted average carbon footprint of all goods classified as brown and green within each product category. This yields the carbon footprint f_{ij}, where $j \in \{b, g\}$ indicates the brown or green variety of product category i. Table 1 summarizes the classification and the resulting average carbon footprints f_{ij}. Note that electricity is not split into a brown or green variety, as this product category comprises only one product.

5.3.2 Utility Function

We need to calibrate a total of 48 parameters of the utility function. Of these, 12 parameters are calibrated externally based on standard values from the literature, while the remaining 36 parameters are calibrated internally using a minimum distance estimation approach. The internally calibrated parameters consist of $I = 9$ CES weights on the brown variety of each good (Ω_{ib}) in the lower-tier utility function, $I = 8$ taste shifters (Ω_i), and $I = 9$ expenditure elasticities (ϵ_i) in the upper-tier utility function.

Externally Calibrated Parameters. We fix the Frisch elasticity of labor supply at $1/\varphi = 0.5$, in line with empirical estimates (Chetty, Guren, Manoli, and Weber 2011). We set the

[18] Table 5 in Appendix B.1 contains the classification of all 110 products into 'green' and 'brown' varieties.

Product Category	Median Carbon Footprint	Number of Goods		Average Carbon Footprint	
		Brown	Green	Brown	Green
Durables	0.2451	20	13	0.3259	0.0525
Electricity	1.4604	–	–	–	–
Food	0.2296	9	6	0.2417	0.1814
Heating	1.2191	2	2	1.6493	0.6321
Housing	0.1031	6	2	0.1275	0.1021
Other	0.0003	4	1	0.2891	0.0002
Services	0.0884	24	11	0.1051	0.0525
Transport	1.9436	1	6	1.9436	0.3981
Vacation	0.4756	1	1	0.4756	0.1468

Table 1: Carbon Footprint of Brown and Green Varieties

Notes: The carbon footprint is measured in kgCO2 per euro of expenditure. Electricity is not divided into a brown and green varieties, as this category contains only one product. The median carbon footprint for each category is computed as the expenditure-weighted median across all products in that category. Average carbon footprints for 'brown' and 'green' varieties are calculated as the expenditure-weighted averages within each group. The column number of goods refers to the number of products classified as either 'brown' or 'green' variety within a category.

coefficient of relative risk aversion to $\gamma = 0.5$ and the elasticity of substitution across products is set to $\sigma = 0.2$. Finally, we assume that the elasticity of substitution between 'brown' and 'green' varieties is identical across all products and set to $\sigma_i = 1.5$. Table 2 summarizes the set of externally calibrated parameters.[19]

Parameter	Description	Value
	Utility Function	
$1/\varphi$	Frisch elasticity of labor supply	0.5
γ	Coefficient of relative risk aversion	0.5
σ	Elasticity of substitution between products	0.2
$\sigma_i \ \forall i$	Elasticity of substitution between varieties	1.5
	Tax-Transfer System	
λ	Overall level of taxation	0.25
τ	Tax progressivity	0.07
m	Transfers given to households with zero income	0.1
ζ	Transfer phase-out rate	8.4
$\bar{y}$	Average gross labor income (in €)	40 359

Table 2: Externally Calibrated Parameters

Notes: The table reports the set of externally calibrated parameters used in the model. The tax-transfer system parameters are chosen to match the German tax-transfer system in 2018, based on data from the LESt and EVS.

[19]The chosen values for γ, σ and φ allow the model to replicated reasonable values of the income effect $\eta(\theta)$. Figure 6 in Appendix B.3.1 illustrates the income effect implied by the model. In Section 6.2 we provide robustness for these parameter choices.

Internally Calibrated Parameters. We calibrate the remaining parameters of the utility function in two steps. First, we calibrate the CES weights on the 'green' and 'brown' varieties Ω_{ij} of the lower-tier utility function based on expenditure shares from the EVS. Specifically, we normalize the CES weight on the 'green' variety to one without loss of generality, i.e. $\Omega_{ig} = 1 \ \forall i$. The corresponding weight in the brown variety Ω_{ib} is then set such that the expenditure shares implied by the CES demand system match the respective shares in the EVS data. Second, we estimate the taste shifter $\mathbf{\Omega} = (\Omega_1, \ldots, \Omega_I)$ and expenditure elasticities of demand $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_I)$ of the upper-tier utility function to match empirical patterns of household expenditure along the gross labor income distribution. We apply a minimum distance estimation approach targeting the (i) the average expenditure share and (ii) the difference in expenditure share between the top and bottom income quartile for each product $i \in \{1, \ldots, I\}$. These empirical moments are computed using household expenditure data from the EVS. The internally calibrated parameters are reported in Table 3. See Appendix B.3.2 for details on the internal calibration strategy.

Model Fit. Figure 1 illustrates the expenditure shares on the 'brown' and 'green' variety across income quartiles for each of the nine products. The solid lines represent the results from the model, while the dashed lines are the corresponding data/empirical moments from the EVS. The red lines correspond to the 'brown' varieties, and the green lines to the 'green' variety. The model captures both the overall levels and the distributional patterns of expenditures across income goods well. Figure 2 displays the average carbon emission intensity per Euro of consumption expenditure. The solid red line depicts the model-implied carbon emission intensity, and the gray dots represent the corresponding empirical moments from the EVS.[20] Overall, the model provides a good fit to both the consumption allocation patterns and the carbon emission intensity along the income distribution.

5.3.3 Tax-Transfer System

We adopt a parametric specification for the tax-transfer system proposed by Ferriere, Grübener, Navarro, and Vardishvili (2023). Total tax payments are given by

$$\mathcal{T}(y) = \exp\left[\log(\lambda)(y/\bar{y})^{-2\tau}\right] y - \mathcal{T}_0(y),$$

where $\bar{y}$ denotes average gross labor income. The first term specifies a two-parameter tax schedule: the parameter λ governs the overall level of taxation and the parameter τ controls

[20] The average emission intensity per Euro of consumption expenditure is obtained by dividing the model-implied total carbon footprint by total consumption expenditures, e.g. $\sum_{i,j} f_{ij} \cdot c_{ij} / \sum_{i,j} c_{ij}$., where f_{ij} is the average emission intensity and c_{ij} the model implied consumption of each subgood ij.

Parameter	Product	Value	Moment	Model	Data
			CES Weight		
Ω_{1b}	Durables	1.07	Expenditure Share on Brown Variety	0.53	0.53
Ω_{2b}	Food	1.02	Expenditure Share on Brown Variety	0.51	0.51
Ω_{3b}	Heating	1.25	Expenditure Share on Brown Variety	0.58	0.58
Ω_{4b}	Housing	7.95	Expenditure Share on Brown Variety	0.96	0.96
Ω_{5b}	Other	1.38	Expenditure Share on Brown Variety	0.62	0.62
Ω_{6b}	Services	1.12	Expenditure Share on Brown Variety	0.54	0.54
Ω_{7b}	Transport	1.21	Expenditure Share on Brown Variety	0.57	0.57
Ω_{8b}	Vacation	2.89	Expenditure Share on Brown Variety	0.83	0.83
Ω_{9b}	Electricity	1	-	-	-
			Expenditure Elasticity of Demand		
ϵ_1	Durables	1.34	Difference in Expenditure Share (Q4 - Q1)	0.075	0.075
ϵ_2	Food	0.85	Difference in Expenditure Share (Q4 - Q1)	-0.061	-0.061
ϵ_3	Heating	0.70	Difference in Expenditure Share (Q4 - Q1)	-0.017	-0.017
ϵ_4	Housing	0.86	Difference in Expenditure Share (Q4 - Q1)	-0.077	-0.077
ϵ_5	Other	1.03	Difference in Expenditure Share (Q4 - Q1)	-0.004	-0.004
ϵ_6	Services	1.35	Difference in Expenditure Share (Q4 - Q1)	0.079	0.079
ϵ_7	Transport	1.12	Difference in Expenditure Share (Q4 - Q1)	-0.002	-0.002
ϵ_8	Vacation	1.76	Difference in Expenditure Share (Q4 - Q1)	0.013	0.013
ϵ_9	Electricity	0.76	Difference in Expenditure Share (Q4 - Q1)	-0.007	-0.007
			Taste Shifter		
Ω_1	Durables	1	Average Expenditure Share	0.255	0.272
Ω_2	Food	10.31	Average Expenditure Share	0.145	0.131
Ω_3	Heating	5.93	Average Expenditure Share	0.026	0.023
Ω_4	Housing	624.99	Average Expenditure Share	0.193	0.175
Ω_5	Other	0.91	Average Expenditure Share	0.026	0.025
Ω_6	Services	1.01	Average Expenditure Share	0.267	0.285
Ω_7	Transport	1.07	Average Expenditure Share	0.061	0.061
Ω_8	Vacation	0.02	Average Expenditure Share	0.014	0.017
Ω_9	Electricity	0.50	Average Expenditure Share	0.013	0.011

Table 3: Internally Calibrated Parameters

Notes: The CES weight on the 'brown' variety of each product is inferred from the observed average expenditure share on the 'brown' variety across the population. Thus, the model exactly matches the corresponding data moment by construction. The moment for the expenditure elasticity of demand is the difference in expenditure shares between the top (Q4) and bottom income quartile (Q1) for each product $i = 1 \ldots, I$. The moment for the taste shifter is the average expenditure share for each product $i = 1, \ldots, I$ across the population. The taste shifter for durables is normalized to one without loss of generality. All data moments are computed using consumption expenditure data form the EVS.

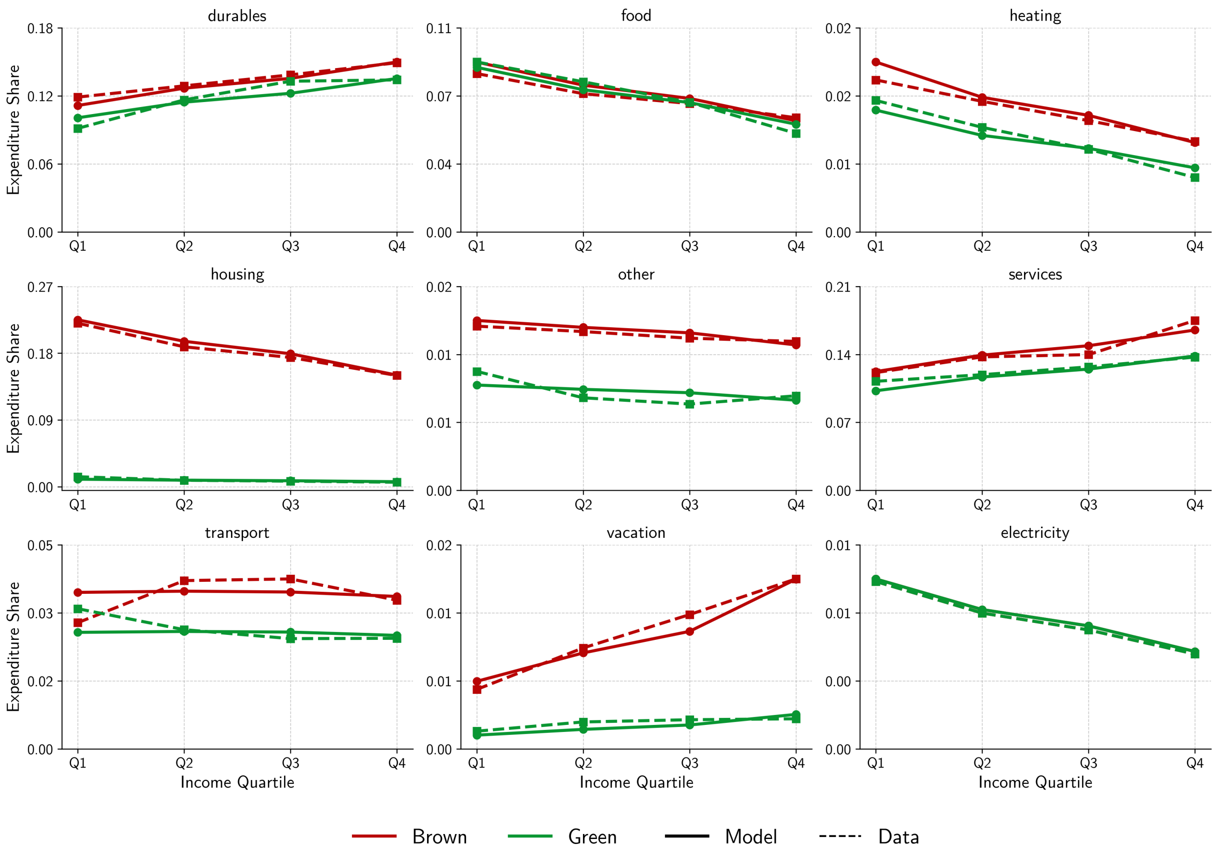


Figure 1: Model Fit (Expenditure Shares)

Notes: The figure shows the model implied expenditure share (solid line) for the 'brown' (red) and 'green' (green) variety of each of the nine products. This is compared to the dashed line which shows the expenditure shares in the EVS data.

the degree of progressivity. The transfer function $\mathcal{T}_0(y)$ phases out with labor income and is given by

$$\mathcal{T}_0(y) = m\bar{y}\frac{2\exp\left[-\zeta\frac{y}{\bar{y}}\right]}{1+\exp\left[-\zeta\frac{y}{\bar{y}}\right]},$$

where m captures the size of transfers given to households with zero income as a share/multiple of average labor income $\bar{y}$ and ζ determines the phase-out rate of transfers. This specification allows us to jointly fit the tax payments of the bottom and the top of the income distribution.

We estimate the tax parameters (λ, τ) using tax return data from the LESt, and the transfer parameters (m, ζ) using data on transfers from the EVS. We estimate the parameters using non-linear least squares. Table 2 presents the estimated parameters and Figure 3(a) illustrates the marginal rates obtained from the estimated tax-transfer function. The solid red line depicts the statutory marginal income tax rate, while the dashed gray line shows the transfer phase-out rate. Transfers decline steeply with gross labor income and are fully phased out at an annual gross labor income of around €40 000. The effective marginal income tax rate is the sum of the statutory marginal income tax rate and the transfer phase-out rate. As a result, the effective marginal tax rate exhibits a sharp drop at the point where transfers are fully phased out, after

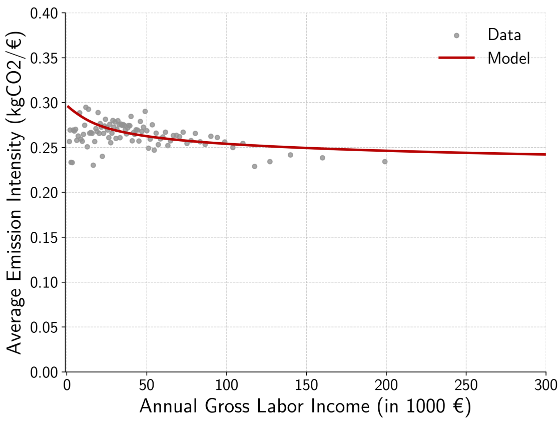


Figure 2: Model Fit (Emission Intensity)

Notes: The figure shows the model implied average emission intensity (in kgCO2) per Euro of consumption expenditure (solid red line). The corresponding empirical moment from the EVS (grey dots). The x-axis shows gross labor income y of a tax unit.

which it converges to the statutory marginal tax rate. Appendix B.4 provides further details on the estimation process.

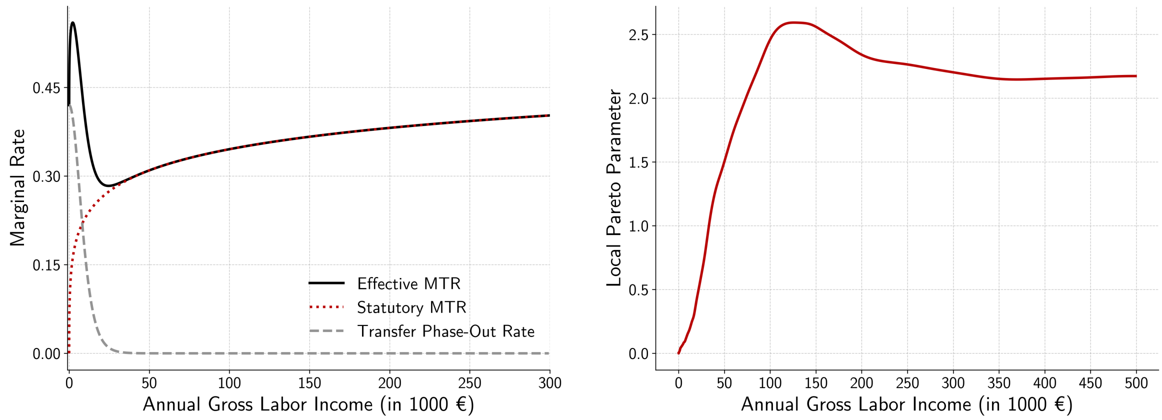
5.3.4 Income Distribution

To calibrate the distribution of gross labor income, we use administrative income tax records from the LESt for the year 2018. Gross labor income is defined as the sum of income from employment and self-employment. We estimate the income distribution $\tilde{h}(y)$ non-parametrically using a standard kernel density estimation. Figure 3(b) illustrates the Local Pareto parameter $\tilde{h}(y)y/(1-\tilde{H}(y)$ derived from the estimated estimated income distribution. At the top, the income distribution $\tilde{h}(y)y/(1-\tilde{H}(y)$ roughly converges consistent with a Pareto tail with a Pareto parameter of around 2. Appendix B.5 documents the estimation procedure in detail.

Given the estimated income distribution and the current German tax-transfer system, we infer the skill distribution $h(\theta)$ by inverting the labor supply first-order condition, following Saez (2001).

6 Quantitative Policy Analysis

We now analyze the optimal carbon rebates and the implications for redistribution in the German case. Our calibrated economy from Section 5 describes the status-quo scenario. Our theoretical comparative statics analysis from Section 4 allows us to answer the following question: How should the German government implement a certain carbon cap goal assuming that the welfare function is unchanged. In our benchmark analysis, we set $\mathcal{G}(x) = x$. The implied social marginal utilities and Pareto weight are illustrated in Figure 8 in Appendix B.6.



(a) Tax-Transfer System (b) Income Distribution

Figure 3: Tax-Transfer System and Income Distribution

Notes: The left panel illustrates the statutory marginal income tax rate (dotted red line) and the transfer phase-out rate (dashed gray line) based on our estimates of the parametric function for the tax-transfer system proposed by Ferriere, Grübener, Navarro, and Vardishvili (2023). The effective marginal income tax rate (solid black line) is the sum of the statuary marginal income tax rate and the transfer phase-out rate. The right panel displays the local Pareto parameter $h_y(y)y/(1 - H_y(y))$, where $H_y(y)$ is the cumulative distribution function and $h_y(y)$ is the probability density function of the gross labor income distribution. The density is estimated using a standard kernel density estimation. The x-axis shows gross labor income y of a tax unit.

6.1 Benchmark Scenario

As a benchmark scenario, we consider the introduction of a carbon cap that achieves a 10% reduction in aggregate carbon emissions relative to the status quo.

Figure 4 illustrates the schedule of the optimal carbon rebate (red solid line) as a function of gross labor income. The rebate increases from around €1 000 for the lowest income tax units to over €6 000 for those earning €300 000. To assess the distributional pattern of the rebate policy, we introduce a *regressivity ratio*, defined as the ratio of the rebate received by the 90th percentile (€92 240 annual gross labor income) to that received by the 10th percentile of the income distribution (€6 945 annual gross labor income). Formally,

$$RR_{90/10}^{\text{rebate}} \equiv \frac{R_{\text{CO}_2}(y_{90})}{R_{\text{CO}_2}(y_{10})},$$

where y_p denotes the income that corresponds to percentile p. For the optimal rebate schedule, we find $RR_{90/10}^{\text{rebate}} = 2.74$, indicating that the tax units in the 90th percentile receive a 2.74 times higher rebate than those in the 10th percentile. This implies that the rebate policy in isolation is regressive. In contrast, the *progressivity ratio* of the carbon tax is $PR_{90/10}^{\text{carbon tax}} = 7.19$, indicating that the 90th percentile pays 7.19 times more in carbon taxes than the 10th percentile.

Hence, despite the regressivity of the carbon rebate schedule, the carbon policy as a whole is strongly progressive. We illustrate this in Figure 5. Figure 5(a) shows the effective marginal tax rate along the gross income distribution. The optimal carbon policy (red solid line) increases

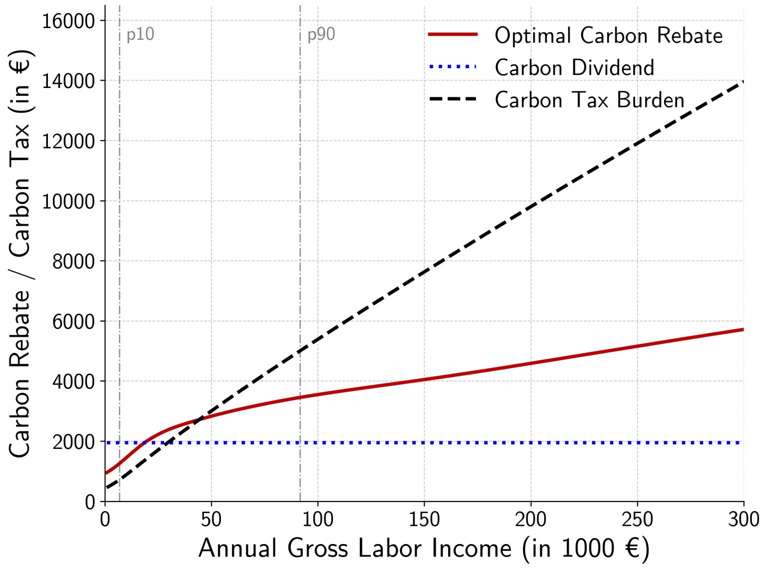


Figure 4: Optimal Carbon Rebate

Notes: The figure shows the optimal carbon rebate as a function of gross household income (red solid line) and the rebate under a carbon dividend policy (black dashed line). The blue dotted line illustrates the carbon tax burden. The gray vertical dashed-dotted lines correspond to the 10th and 90th percentile of the annual gross labor income distribution.

effective marginal tax rates relative to the status quo (black dashed line) for incomes above €18 000. For the highest incomes, this increase exceeds 4.5 percentage points.

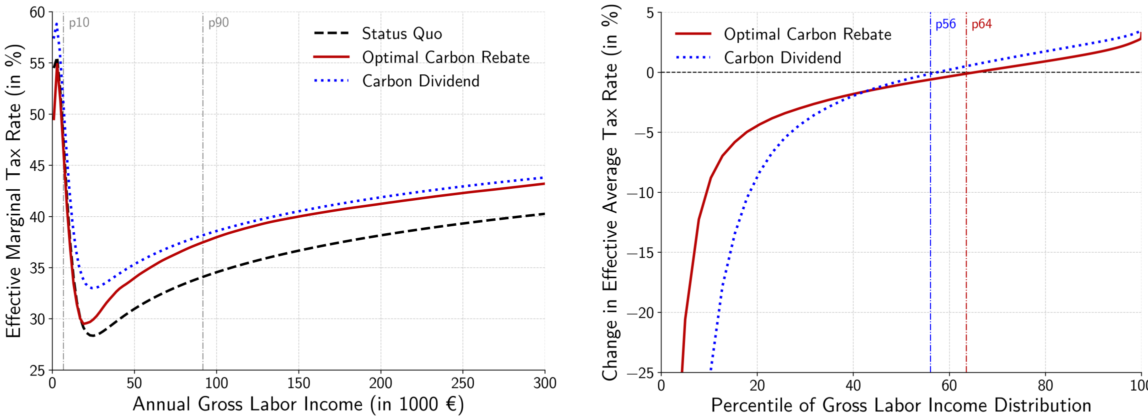
Since the effective marginal tax rates do not account for the lump-sum component, we consider changes in effective average tax rates in Figure 5(b). The red solid line illustrates the net burden of the carbon policy, defined in the change in the effective average tax rate, as a function of the percentile of the income distribution. The net burden rises from approximately −40 percentage points at the very bottom bottom to 3.25 percentage points at the top of the distribution. Further, tax units until the 64th percentile are net beneficiaries of the carbon policy, receiving more in rebates than they pay in carbon taxes.

To quantify the change in redistribution in one metric that we can compare across scenarios, we consider the difference in the effective average tax rate (EATR) between the 90th and the 10th percentile. Formally, we define this difference as

$$\Pi^{x}_{90/10} \equiv \bar{\tau}^{x}_{eff}(y_{90}) - \bar{\tau}^{x}_{eff}(y_{10}) \quad x \in \{sq, co2\},$$

where $\bar{\tau}^{x}_{eff}(y_p)$ denotes the EATR of percentile p under the status quo and the optimal carbon policy. To measure the change in redistribution, we consider the change in the EATR gap

$$\Delta\Pi_{90/10} \equiv \Pi^{co2}_{90/10} - \Pi^{sq}_{90/10},$$



(a) Effective Marginal Tax Rates (b) Net Burden of Carbon Policy

Figure 5: Redistributive Implications of Carbon Policy

Notes: The right panel shows the effective marginal tax rate τ_{eff} as a function of gross labor income. The gray vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution. The left panel shows the change in the effective average tax rate $\bar{\tau}_{eff}$ across percentiles of the gross labor income distribution, as defined in (15). The vertical dashed-dotted line indicates the percentile with no change in the effective average tax rate. The black dashed line represents the status quo without carbon policies, the red solid line the optimal carbon rebate policy, and the blue dotted line the carbon dividend policy.

where $\Pi^{co2}_{90/10}$ and $\Pi^{sq}_{90/10}$ denote the EATR gap between the top and bottom deciles under the carbon policy and the status quo, respectively. A positive value of $\Delta\Pi_{90/10}$ indicates an increase in redistribution.

Under the status quo, $\bar{\tau}^{sq}_{eff}(y_{90}) = 28.94\%$ and $\bar{\tau}^{sq}_{eff}(y_{10}) = -5.66\%$, yielding an EATR gap of $\Pi^{sq}_{90/10} = 34.59$ percentage points. Under the optimal policy, these values change to $\bar{\tau}^{co2}_{eff}(y_{90}) = 30.56\%$ and $\bar{\tau}^{co2}_{eff}(y_{10}) = -14.46\%$ respectively, implying an EATR gap of $\Pi^{co2}_{90/10} = 45.01$ percentage points. The resulting change of $\Delta\Pi_{90/10} = 10.42$ percentage points reflects substantial rise in redistribution under the optimal carbon policy.

Comparison to Carbon Dividend. We compare the optimal carbon rebate to a carbon dividend policy that achieves the same aggregate emissions reduction. The carbon dividend is illustrated by the blue dotted line in Figure 4. Compared to the optimal policy, it implies more generous rebates for the bottom 26% of the income distribution and less generous benefits for the top 74%. Hence, a majority would prefer the optimal policy over the carbon dividend in our model.

An alternative question is whether the carbon dividend or the optimal policy would receive majority support compared to the situation with no carbon policies. For the carbon dividend, only tax units up to the 56th percentile are net beneficiaries from the overall carbon policy (see Figure 5(b)), so there would be a close majority. For the optimal policy, this number is 64%.

We also illustrate the implied effective marginal tax rates under the carbon dividend policy are illustrated by the blue dotted line in Figure 5(a). One can clearly see the increase in the effective marginal tax rates compared to the optimal policy. For example, for incomes below €25 000, effective marginal tax rates are between 3 and 8 percentage points higher in the carbon dividend scenario. The implied increase in redistribution is significantly higher with $\Delta\Pi_{90/10} = 27.26$.

6.2 Robustness Analysis

We now consider five alternative parameter specifications, with the results summarized in Table 4.[21] Overall, the findings are highly robust. The optimal steepness of the rebate schedule varies slightly across specifications, consistent with the insights from the theoretical analysis. The same holds for the implications regarding the increase in redistribution.

Case	$RR^{\text{rebate}}_{90/10}$	$PR^{\text{carbon tax}}_{90/10}$	$\Delta\Pi_{90/10}$	τ_{co_2}	% Beneficiaries
Baseline	2.74	7.19	10.42 pp	336	64%
Lower elasticity ($\varphi = 0.25$)	3.86	7.37	6.97 pp	388	70%
Higher Curvature parameter ($\gamma = 1.5$)	4.24	7.48	6.24 pp	382	70%
Lower brown green substitution ($\sigma_i = 0.5\ \forall\ i$)	2.68	6.62	19.96 pp	660	64%
Curvature in welfare function ($\gamma_{\mathcal{G}} = 3$)	4.58	7.50	4.53 pp	365	70%

Table 4: Robustness of Policy Implications

Notes: The table reports distributional patterns of the optimal rebate policy for the baseline specification and five alternative parameter settings. The case *pro rich brown green substitution* refers to a scenario in which the elasticity of substitution between the brown and green varieties differs across consumption categories, allowing richer households to substitute more easily from brown to green varieties. The carbon tax, τ_{co2}, is measured in tCO2/€ and the change in the EATR gap, $\Delta\Pi_{90/10}$, in percentage points (pp). The column *% Beneficiaries* reports the income percentile threshold below which tax units are net beneficiaries of the overall carbon policy.

Lower Elasticity. In this scenario, we set the Frisch elasticity parameter to half its benchmark value, i.e. $\varphi = 0.25$. The optimal rebate becomes more regressive, with our regressivity measure increasing from 2.74 to 3.86. The progressivity ratio of the carbon tax is almost unchanged. Consequently, the increase in redistribution of the carbon policy is smaller: the EATR gap rises by only 6.97 percentage points.

At first sight, this may seem surprising, as a lower elasticity typically implies higher optimal taxes and thus more redistribution. However, in our setup, lowering the elasticity also changes the structure of Pareto weights in such a way that the German tax-transfer system of 2018 remain optimal. For this to hold, the Pareto weights must become less 'pro-poor', offsetting the potential redistribution gains from higher optimal taxes.

Higher Curvature. In this scenario, we increase the curvature of consumption utility to $\gamma = 1.5$. The optimal rebate again becomes more regressive. As the government expands redistribution, the gains from redistribution (as captured by the distributional gains term) diminish

[21] We document the optimal carbon rebate schedule, the effective marginal tax rates and the change in the effective average tax rates along the gross labor income distribution for all five specifications in Appendix B.7.

more quickly than in the benchmark case with a lower curvature. Consequently, it is optimal to increase redistribution by less, and the EATR gap amounts to only 7.48 percentage points. The change in the curvature also affects income effects. Below, we consider an alternative scenario in which these income effects are held constant.

Lower Elasticity of Substitution between Brown and Green Varieties. In this scenario, a substantially higher carbon tax is required to achieve the same reduction in the aggregate carbon footprint. The regressivity of the optimal rebate is slightly smaller relative to the benchmark. The increase in redistribution, $\Delta\Pi_{90/10}$, is much larger, however, this is primarily driven by the fact that both the level of the rebate and the carbon tax rate are considerably higher in this scenario.

Curvature in Welfare Function. We assume $\mathcal{G}(x) = \frac{x^{1-\gamma_{\mathcal{G}}}}{1-\gamma_{\mathcal{G}}}$ with $\gamma_{\mathcal{G}} = 3$ for the calibration of the inverse-optimum weights. This substantial increase in curvature implies a markedly smaller increase in redistribution and a rebate schedule that rises more steeply with income. The effect is similar to the second robustness check with $\gamma = 1.5$, but here the curvature effect on the $g(\theta, \mathcal{P})$ is isolated because income effects are held constant.

7 Further Aspects

7.1 Endogenous Emission Intensities

We have so far treated emission intensities f_{ij} as exogenous. We conjecture that this assumption is not crucial for our theoretical results. Following van der Ploeg, Rezai, and Reanos (2022) and consistent with the DICE model (Nordhaus and Sztorc 2013), one could instead introduce convex carbon abatement costs, which would render emission intensities endogenous to the level of the carbon tax. Endogenizing f_{ij} would reduce the effective carbon tax burden on households, since higher carbon prices would induce abatement and thereby dampen the increase in gross consumer prices. Quantitatively, this would imply lower carbon tax burdens and correspondingly smaller rebates. However, our main mechanism remains unchanged: in the presence of carbon emissions, producing output is socially less desirable.

7.2 Innovation Sector

Our finding that carbon emissions reduce the social value of labor supply has a degrowth flavor to it. As mentioned in the introduction, a key reason why mainstream economists reject the degrowth idea is that innovation can reduce the carbon footprint. This raises the question whether our findings are robust to the existence of 'green' innovation.

If the carbon footprint of output has been lowered due to green innovation, the social value of output and, by implication, the social value of supplying labor increases, so that the efficiency cost of redistribution rises. But for a given carbon footprint, the case for more redistribution compared to the case where global warming does not exist continues to hold. The result remains relevant as long as innovation has not led to climate neutrality.

Another concern regarding the generality of our result is that policies which discourage labor supply may also discourage innovation. In case that no other market imperfections exist, the existence of the carbon price makes sure that the return to 'green' innovation is sufficient and the allocation of labor to the green innovation sector should not be affected by the increase in redistribution.

Yet, there are indeed various reasons why underinvestment in green innovation may arise. First, research and development tend to generate positive spillovers in the form of knowledge spillovers. This may justify subsidies for research and development. Second, investors may not be convinced that governments will stick to carbon price paths they have announced. If this leads to underinvestment in green innovation, it may be more difficult for governments to raise carbon prices as announced. This type of commitment problem may justify up front subsidies for investment in green innovation.

In any case, to make sure that sufficient incentives to innovate exist, governments should then use targeted instruments to allocate more labor to the green innovation sector, rather than change policies affecting labor supply generally.

8 Conclusion

In this paper we study the interaction between carbon taxation and redistribution. A comparative statics analysis shows that the introduction of a carbon cap increases the optimal level of redistribution, independent of the social welfare function. However, the optimal increase in redistribution is smaller than what a carbon dividend would imply. Implementing the new desired allocation through carbon tax rebates requires rebates rise with income, a result that holds independently of the welfare function. Only in a knife-edge case, requiring, among other assumptions, constant social marginal utility of consumption, is it optimal to implement a lump-sum carbon tax rebate.

We quantify our model using German administrative and household microdata, matched with environmentally extended input-output tables to estimate carbon footprints as a function of income. Our results show that the optimal rebate is roughly three times larger for households at the 90th percentile than for those at the 10th percentile of the income distribution. While such a rebate schedule appears regressive, the overall policy of carbon taxation combined with carbon tax rebates significantly increases redistribution: carbon tax payments are about seven times higher at the 90th percentile than at the 10th percentile of the income distribution.

The optimal policy therefore makes more households better off than a carbon dividend would, suggesting that it could also enjoy greater political support.

References

Aghion, P., T. Boppart, M. Peters, M. Schwartzman, and F. Zilibotti (2025): "A theory of endogenous degrowth and environmental sustainability," Discussion paper, National Bureau of Economic Research.

Ahlvik, L., M. Liski, and J. Mäkimattila (2025): "Pigouvian Income Taxation," .

Atkinson, A., and J. Stiglitz (1976): "The design of tax structure: Direct versus indirect taxation," *Journal of Public Economics*, 6(1), 55–75.

Barrage, L. (2020): "Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy," *The Review of Economic Studies*, 87(1), 1–39.

Bierbrauer, F. (2025): "Is a Uniform Price on Carbon Desirable?," .

Bovenberg, A. L., and R. A. De Mooij (1994): "Environmental levies and distortionary taxation," *The American Economic Review*, 84(4), 1085–1089.

Bovenberg, A. L., and F. van der Ploeg (1994): "Environmental policy, public finance and the labour market in a second-best world," *Journal of Public Economics*, 55(3), 349–390.

Bundesagentur für Arbeit (2018): "Ausgaben für aktive und passive Leistungen im SGB II (Jahreszahlen)," .

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive mMrgins," *The American Economic Review*, 101(3), 471–475.

Comin, D., D. Lashkari, and M. Mestieri (2017): "Structural Change With Long-Run Income and Price Effects," *Working Paper*.

——— (2021): "Structural Change With Long-Run Income and Price Effects," *Econometrica*, 89(1), 311–374.

Douenne, T., A. J. Hummel, and M. Pedroni (2025): "Optimal Fiscal Policy in a Climate-Economy Model with Heterogeneous Households," *Working Paper*.

Familienkasse Direktion SR1 (2018): "Kindergeld / Kinderzuschlag Jahreszahlen 2018," .

Ferey, A., B. B. Lockwood, and D. Taubinsky (2024): "Sufficient statistics for nonlinear tax systems with general across-income heterogeneity," *American Economic Review*, 114(10), 3206–3249.

Ferriere, A., P. Grübener, G. Navarro, and O. Vardishvili (2023): "On the Optimal Design of Transfers and Income Tax Progressivity," *Journal of Political Economy Macroeconomics*, 1(2), 276–333.

German Statistical Office (2023): "Umweltoekonomische Gesamtrechnungen (UGR) Private Haushalte und Umwelt - Berichtszeitraum 2000-2021," .

——— (2024a): "VGR des Bundes - Konsumausgaben der privaten Haushalte," .

——— (2024b): "Wohngeld - Ausgaben insgesamt nach Bundesländern im Zeitvergleich am 31.12.," .

——— (2025a): “Bruttoausgaben, Einnahmen, Nettoausgaben der Sozialhilfe: Deutschland, Jahre, Sozialhilfearten (22111-0003),” .

——— (2025b): “CO2 footprint of private households by areas of demand,” .

Golosov, M., M. Graber, M. Mogstad, and D. Novgorodsky (2023): “How Americans Respond to Idiosyncratic and Exogenous Changes in Household Wealth and Unearned Income,” *The Quarterly Journal of Economics*, 139(2), 1321–1395.

Goulder, L. H. (1995): “Environmental taxation and the double dividend: a reader’s guide,” *International tax and public finance*, 2, 157–183.

Hänsel, M. C., M. Franks, M. Kalkuhl, and O. Edenhofer (2022): “Optimal carbon taxation and horizontal equity: A welfare-theoretic approach with application to German household data,” *Journal of Environmental Economics and Management*, 116, 102730.

Hardadi, G., A. Buchholz, and S. Pauliuk (2021): “Implications of the distribution of German household environmental footprints across income groups for integrating environmental and social policy design,” *Journal of Industrial Ecology*, 25(1), 95–113.

Heathcote, J., and H. Tsujiyama (2021): “Optimal income taxation: Mirrlees meets Ramsey,” *Journal of Political Economy*, 129(11), 3141–3184.

Jacobs, B. (2018): “The marginal cost of public funds is one at the optimal tax system,” *International Tax and Public Finance*, 25(4), 883–912.

Jacobs, B., and R. A. De Mooij (2015): “Pigou meets Mirrlees: On the irrelevance of tax distortions for the second-best Pigouvian tax,” *Journal of Environmental Economics and Management*, 71, 90–108.

Jaravel, X., and A. Olivi (2024): “Prices, Non-homotheticities, and Optimal Taxation,” .

Kaplow, B. L. (2012): “Optimal control of externalities in the presence of income taxation,” *International Economic Review*, 53(2), 487–509.

Meadows, D. H., D. H. Meadows, J. Randers, and W. W. Behrens III (1972): “The limits to growth: a report to the club of Rome (1972),” 91(2).

Miller, R. E., and P. D. Blair (2009): *Input-Output Analysis: Foundations and Extensions*. Cambridge University Press, Cambridge, UK, 2nd edn.

Mirrlees, J. A. (1971): “An Exploration in the Theory of Optimum Income Taxation12,” *The Review of Economic Studies*, 38(2), 175–208.

Nordhaus, W., and P. Sztorc (2013): “DICE 2013R: Introduction and user’s manual,” .

Nordhaus, W. D., R. N. Stavins, and M. L. Weitzman (1992): “Lethal model 2: The limits to growth revisited,” *Brookings papers on economic activity*, 1992(2), 1–59.

Oswald, Y., A. Owen, and J. K. Steinberger (2020): “Large Inequality in International and Intranational Energy Footprints between Income Groups and across Consumption Categories,” *Nature Energy*, 5, 231 – 239.

Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) (2018a): “Einkommens- und Verbrauchsstichprobe (HB) 2018, On-Site-Zugang,” .

——— (2018b): "Einkommens- und Verbrauchsstichprobe (NGT) 2018, On-Site-Zugang," .

——— (2018c): "Lohn- und Einkommenssteuerstatistik 2018, Gastwissenschaftsarbeitsplatz," .

Saez, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *The Review of Economic Studies*, 68(1), 205–229.

——— (2002): "The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes," *Journal of Public Economics*, 83(2), 217–230.

Savin, I., and J. van den Bergh (2024): "Reviewing studies of degrowth: Are claims matched by data, methods and policy analysis?," *Ecological Economics*, 226, 108324.

Stadler, K., R. Wood, T. Bulavskaya, C.-J. Södersten, M. Simas, S. Schmidt, A. Usubiaga, J. Acosta-Fernández, J. Kuenen, M. Bruckner, S. Giljum, S. Lutter, S. Merciai, J. H. Schmidt, M. C. Theurl, C. Plutzar, T. Kastner, N. Eisenmenger, K.-H. Erb, A. de Koning, and A. Tukker (2018): "EXIOBASE 3: Developing a Time Series of Detailed Environmentally Extended Multi-Regional Input-Output Tables," *Journal of Industrial Ecology*, 22(3), 502–515.

Statistisches Bundesamt (2025): "Greenhouse gas emissions1 (residents concept)," .

Steen-Olsen, K., R. Wood, and E. G. Hertwich (2016): "The Carbon Footprint of Norwegian Household Consumption 1999–2012," *Journal of Industrial Ecology*, 20(3), 582–592.

The Wall Street Journal (2019): "Economists' Statement on Carbon Dividends," *The Wall Street Journal*.

Tisserant, A. (2016): "Concordance from COICOP to ISIC Rev3," .

Umweltbundesamt (2025): "Emissionen sinken 2018 um mehr als 31 Prozent gegenüber 1990," .

van der Ploeg, F., A. Rezai, and M. T. Reanos (2022): "Gathering support for green tax reform: Evidence from German household surveys," *European Economic Review*, 141, 103966.

World Bank (2025): "Carbon Pricing Dashboard," Accessed: 2025-08-12.

A Theoretical Appendix

A.1 Proof of Lemma 1

Recall the first-order condition for labor supply (4):

$$u_e(\theta, \mathcal{P})\theta\left(1 - \mathcal{T}'(y(\theta))\right) = \frac{y(\theta)}{\theta}^{\frac{1}{\varphi}}. \tag{30}$$

We proof the result by showing that both tax reforms affect the left-hand side (LHS) in the same way.

Carbon Tax Reform. A marginal increase in the carbon tax by $d\tau_{co_2}$ affects the optimality condition through $u_e(\theta, \mathcal{P})$. Denote its change due to the carbon tax by $du_e(\theta)$. It is given by:

$$\begin{aligned} du_e(\theta) =& \frac{\partial u_e(\theta)}{\partial \tau_{co_2}} d\tau_{co_2} = \frac{\partial \left(u_{\tau_{co_2}} d\tau_{co_2}\right)}{\partial e} \\ =& \frac{\partial \left(-u_e(\theta) \sum_{i=1}^{n} f_i c_i(\theta) d\tau_{co_2}\right)}{\partial e} \\ =& - u_e(\theta) f_e(\theta) d\tau_{co_2} - u_{ee} f(\theta) d\tau_{co_2}. \end{aligned}$$

Hence, the relative change of the LHS of (30) is given by:

$$-f_e(\theta) d\tau_{co_2} - \frac{u_{ee}}{u_e} f(\theta) d\tau_{co_2}. \tag{31}$$

Income Tax Reform. Next, consider an equivalent income tax reform that satisfies

$$d\mathcal{T}(y(\theta)) = f(\theta) d\tau_{co_2},$$

and hence also

$$d\mathcal{T}'(y(\theta)) = \left(1 - \mathcal{T}'(y(\theta)\right) f_e(\theta) d\tau_{co_2}.$$

The change in the LHS of (30) is given by

$$-u_{ee} f(\theta) d\tau_{co_2} \theta \left(1 - \mathcal{T}'(y(\theta))\right) - u_e \theta \left(1 - \mathcal{T}'(y(\theta)\right) f_e(\theta) d\tau_{co_2}.$$

Dividing this expression by $u_e\theta\left(1 - \mathcal{T}'(y(\theta))\right)$ yields that the same relative effect on the LHS of (30) as in the carbon tax case (31).

A.2 Effective marginal tax rate

Total tax revenue raised from a household of type θ is given by

$$\mathcal{T}(y(\theta)) + \frac{t_c}{1+t_c}e(\theta) + \frac{\tau_{co_2}}{1+t_c}f(\theta).$$

We define $\tau_{eff}(\theta, \mathcal{P})$ as the increase in government revenue resulting from a one unit increase in the gross labor income of a household of type θ. Differentiating total tax revenue from a household θ with respect to its gross household income $y(\theta)$ gives

$$\tau_{eff}(\theta, \mathcal{P}) = \mathcal{T}'(y(\theta)) + (1 - \mathcal{T}'(y(\theta))) \left(\frac{t_c}{1+t_c} + \frac{\tau_{co_2}}{1+t_c} \frac{\partial f(\theta)}{\partial e(\theta)} \right),$$

which implies

$$\tau_{eff}(\theta, \mathcal{P}) = \frac{\mathcal{T}'(y(\theta)) + t_c + (1 - \mathcal{T}'(y(\theta)))\, \tau_{co_2} \frac{\partial f(\theta)}{\partial e}}{1+t_c}.$$

A.3 Proof of Lemma 2

The Lagrangian of the government's problem is given by

$$\begin{aligned}\mathcal{L} = &\int_{\underline{\theta}}^{\overline{\theta}} \left(u(e(\theta); \mathbf{p}, t_c, \mathcal{T}(\cdot)) - v(l(\theta))\right) w(\theta) h(\theta) d\theta \\ &+ \lambda \int_{\underline{\theta}}^{\overline{\theta}} \left(\mathcal{T}(y(\theta)) + \frac{t_c}{1+t_c}(y(\theta) - \mathcal{T}(y(\theta)) \right) h(\theta) d\theta - \lambda \mathcal{R},\end{aligned}$$

where $v(\cdot) = \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$ is the disutility of labor effort. Following Saez (2001), consider a small increase in the marginal income tax rate $d\mathcal{T}'$ within a small interval $[y(\theta^*); y(\theta^*) + dy]$. The mass of people affected by this increase in the marginal income tax rate is approximately given by $\tilde{h}(y(\theta^*)) \times dy$, where $\tilde{h}(\cdot)$ is the pdf of the endogenous income distribution defined by $\tilde{H}(\theta^*) = H(y(\theta^*))$. Hence, $h(\theta^*) = \tilde{h}(y(\theta^*))\, y_\theta(\theta^*)$ and

$$\tilde{h}(y(\theta^*)) \times dy = \frac{h(\theta^*) dy}{y_\theta(\theta^*; \mathbf{p}, T(\cdot), t_c)} = \frac{h(\theta^*)\theta^* dy}{\varepsilon_{y,\theta}(\theta^*) y(\theta^*)}.$$

We now collect the three effects that such a reform has on welfare: (i) a substitution effect for those individuals in the interval, (ii) a mechanical tax increase for those to the right of the interval and (iii) an income effect for those same individuals. The sum of the effects must equal zero. We now briefly discuss these three effects which is standard exercise in the literature.

Substitution Effect. Individuals with an income of $y \in [y(\theta^*); y(\theta^*) + dy]$ change their earnings in response to a small change in the marginal tax rate. The change in earnings is given by

$$\frac{\partial y(\theta^*)}{\partial \mathcal{T}'} d\mathcal{T}' = -\varepsilon_{y,1-\mathcal{T}'}(\theta^*) \frac{y(\theta^*)}{1-\mathcal{T}'(y(\theta))} d\mathcal{T}',$$

where $\varepsilon_{y,1-\mathcal{T}(\theta^*)}$ denotes the elasticity of earnings with respect to the retention rate. Multiplying this earnings response by the effective marginal tax rate and the mass of individuals in the intervals yields the substitution effect on welfare

$$dS(\theta^*) = -\Lambda \frac{\tau_{eff}(\theta^*)}{1-\mathcal{T}'(y(\theta^*))} \frac{\varepsilon_{y,1-\mathcal{T}'}(\theta^*)}{\varepsilon_{y,\theta}(\theta^*)} \theta^* d\mathcal{T}' h(\theta^*) dy, \tag{32}$$

which can be simplified to

$$dS(\theta^*) = -\Lambda \frac{\tau_{eff}(\theta^*)}{1-\mathcal{T}'(y(\theta^*))} \frac{\varphi}{1+\varphi} \theta^* d\mathcal{T}' h(\theta^*) dy. \tag{33}$$

Mechanical Effect. Households with $\theta > \theta^*$ pay $d\mathcal{T}'dy$ more taxes leading to a mechanical effect:

$$dM(\theta^*) = d\mathcal{T}'dy \int_{\theta^*}^{\bar{\theta}} \left(\frac{\Lambda}{1+t_c} - g(\theta) \right) dH(\theta). \tag{34}$$

Note that the revenue term is scaled by $\frac{1}{1+t_c}$ because for each dollar of additional income taxes, the government looses $\frac{t_c}{1+t_c}$ in the form of VAT tax revenue.

Income Effect. Households with $\theta > \theta^*$ face a reduction in their income by $d\mathcal{T}'dy$ and thus, adjust their income. The revenue effect of this income effect is given by

$$dI(\theta^*) = d\mathcal{T}'dy\Lambda \int_{\theta^*}^{\bar{\theta}} \tau_{eff}(\theta)\eta(\theta) dH(\theta). \tag{35}$$

Finally, note that the marginal value of public funds is given by[22]

$$\Lambda = \frac{\int_{\underline{\theta}}^{\bar{\theta}} g(x)h(x)dx}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\bar{\theta}} \tau_{eff}(x)\eta(x)h(x)dx}.$$

Optimal Tax Schedule. If the tax schedule is optimal, all three welfare effects must sum to zero: $dS(\theta^*) + dM(\theta^*) + dI(\theta^*) = 0$. Rearranging and inserting the expression for the marginal value of public funds yields then Lemma 2.

[22] The expression follows from a marginal variation of the lump-sum component of the tax schedule.

A.4 Optimal Policies under a Carbon Cap

In the case of the carbon cap constraint, the Lagrangian of the government problem reads as:

$$
\begin{aligned}
\mathcal{L} = &\int_{\underline{\theta}}^{\overline{\theta}} \left(u(e(\theta); \mathbf{p}, t_c, \tau_{co_2}, \mathcal{T}(\cdot)) - v(l(\theta))\right) w(\theta)h(\theta)d\theta \\
&\lambda \int_{\underline{\theta}}^{\overline{\theta}} \left(\mathcal{T}(y(\theta)) + \frac{t_c}{1+t_c}(y(\theta) - \mathcal{T}(y(\theta)) + \frac{\tau_{co_2}}{1+t_c} f(\theta)\right) h(\theta)d\theta - \lambda \mathcal{R} \\
&- \mu \left(\int_{\underline{\theta}}^{\overline{\theta}} f(\theta)dH(\theta) - \bar{F}\right)
\end{aligned}
$$

We first derive the expression for the optimal carbon tax in Appendix A.4.1, and then turn to the characterization of the optimal rebate schedule in Appendix A.4.2.

A.4.1 Proof of Lemma 3

Consider a marginal policy variation consisting of a small increase in the carbon tax by $d\tau_{co_2}$, accompanied by an offsetting reform of the income tax schedule $\mathcal{T}(y(\theta))$ as defined in (7), but with the opposite sign, i.e.

$$\forall \theta \; d\mathcal{T}(y(\theta)) = -f(\theta)d\tau_{co_2}.$$

By Lemma 1, this joint reform leaves labor supply decisions and utility levels of all households unaffected. The only effects on welfare therefore arise through changes in carbon footprints, which operate via the government budget constraint and the carbon cap constraint. These effects sum to

$$\int_{\underline{\theta}}^{\overline{\theta}} f_{\tau_{co_2}}(\theta) \left(\Lambda \frac{\tau_{co_2}}{1+t_c} - \mu\right) dF(\theta).$$

In the optimum, this effect must equal zero, which yields the result in Lemma 3:

$$\tau_{co_2} = (1+t_c)\frac{\mu}{\Lambda}.$$

A.4.2 Proof of Proposition 2

The proof closely follows the proof of Lemma 2 in Appendix A.3 and builds on Proposition 1. There are, however, two differences:

1. Changes in labor supply matter not only through their fiscal externalities but also through their impact on the carbon cap constraint. By Proposition 1, this implies that income changes must be weighted by $\mathrm{T}(\theta)$ rather than by $\tau_{eff}(\theta)$.

2. Even when labor supply is constant, changes in the level of tax payments for households richer than $y(\theta^*)$ can have income effects on their consumption composition and thereby

their carbon footprint. These consumption composition changes, however, do not affect the government's objective: (i) for households themselves, the envelope theorem applies, and (ii) the resulting changes in carbon footprints have no welfare effect at the margin, since the induced change in carbon tax revenue is valued exactly as the corresponding change in the slackness of the carbon cap constraint. This follows from Lemma 3, in the same way that Proposition 1 follows from Lemma 3.

Substitution Effect. The substitution effect corresponding to (33) now takes the form

$$dS(\theta^*) = -\Lambda \frac{\mathrm{T}(\theta^*)}{1-\mathcal{T}'(y(\theta^*))} \frac{\varepsilon_{y,1-\mathcal{T}'}(\theta^*)}{\varepsilon_{y,\theta}(\theta^*)} \theta^* d\mathcal{T}' h(\theta^*) dy., \tag{36}$$

which follows directly from Proposition 1.

Income Effect. Equivalently, the corresponding income effect, analogous to (35), is now given by

$$dI(\theta^*) = d\mathcal{T}' dy \Lambda \int_{\theta^*}^{\overline{\theta}} \mathrm{T}(\theta^*) \eta(\theta) dH(\theta). \tag{37}$$

Mechanical Effect. The mechanical effect $dM(\theta^*)$ remains unchanged compared to (34).

The marginal value of public funds is now given by

$$\Lambda = \frac{\int_{\underline{\theta}}^{\overline{\theta}} g(x)h(x)dx}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\overline{\theta}} \mathrm{T}(x)\eta(x)h(x)dx}. \tag{38}$$

Optimal Tax Schedule. Setting $dS(\theta^*) + dM(\theta^*) + dI(\theta^*) = 0$ and substituting (38) yiels the expression in Proposition 2.

A.4.3 Proof of Proposition 3

We first examine how a distributionally neutral implementation of the carbon cap affects the distributional gains term. Since such a policy utility levels unchanged, only changes in the marginal utility of expenditure can affect the distributional gains term. From the proof of Lemma 1 in Appendix A.1, the change in marginal utility induced by a carbon tax increase is given by:[23]

$$\frac{\partial u_e(\theta)}{\partial \tau_{co_2}} d\tau_{co_2} = -\, u_e(\theta) f_e(\theta) \tau_{co_2} - u_{ee} f(\theta) \tau_{co_2}.$$

[23]Since we consider here an infinitesimal increase of the carbon tax from zero, we have $\tau_{co_2} = d\tau_{co_2}$.

Next, consider the change in the marginal utility due the distributionally neutral rebate policy satisfying $R(y(\theta)) = \tau_{co_2} f(\theta)$:[24]

$$\frac{\partial u_e(\theta)}{\partial R(y(\theta))} dR(y(\theta)) = u_{ee} f(\theta) \tau_{co_2}.$$

Hence, the overall change in marginal utility is given by

$$du_e(\theta) = \frac{\partial u_e(\theta)}{\partial \tau_{co_2}} \tau_{co_2} + \frac{\partial u_e(\theta)}{\partial R(y(\theta))} dR(y(\theta)) = -u_e(\theta) f_e(\theta) \tau_{co_2}.$$

Under Assumption 2, $f_e(\theta) = \alpha \, \forall \theta$, which gives (28).[25]

The remainder of the proof, concerning the efficiency costs term, is provided in the main text.

A.4.4 Proof of Proposition 4

We first consider how the distributional gains term is affected by implementing the carbon cap with a carbon dividend. Under Assumption 2, which implies both linear utility in expenditure $u(e)$ and linear carbon Engel curves, the change in marginal utility simplifies to

$$du_e(\theta) = -u_e(\theta) \alpha \tau_{co_2}. \tag{39}$$

This resembles the result from Appendix A.4.3 for the distributionally neutral carbon rebate policy. Yet, under a carbon dividend there is a crucial difference: while the ratio of marginal utilities remains unchanged, the ratio of welfare weights does not, since utility levels shift. Households that receive more in carbon dividends than they pay in carbon taxes experience an increase in utility, whereas households that pay more than they receive experience a decrease in utility. As a result, inequality in utilities falls, thereby reducing the distributional gains term. To show this formally, we substitute the social marginal utility $g(\theta, \mathcal{P})$ into the expression for the distributional gains term:

$$D(\theta^*, \mathcal{P}_{co_2}) = 1 - \frac{E[\mathcal{G}'(V(\theta, \mathcal{P}))\, \omega(\theta) u_e(\theta, \mathcal{P}) | \theta \geq \theta^*]}{E[\mathcal{G}'(V(\theta, \mathcal{P}))\, \omega(\theta) u_e(\theta, \mathcal{P})]}. \tag{40}$$

Formally, the carbon dividend affects the utility level of type θ by

$$du(\theta) = u_e(\theta) \left(\bar{R}_{co_2} - \tau_{co_2} \alpha e(\theta) \right),$$

[24] Since we consider here an infinitesimal increase of the carbon rebate from zero, we have $R(y(\theta)) = dR(y(\theta))$.

[25] Note that due to the linearity of $u(e; \mathcal{P})$ in e, we could have gotten to this results with fewer steps since $u_{ee} = 0$. Yet, we kept it more general because this result about the distributional gains term holds also for utility functions with curvature in e.

implying that the social marginal utility of type θ changes by

$$d\mathcal{G}'\left(V(\theta,\mathcal{P})\right)=\mathcal{G}''\left(V(\theta,\mathcal{P})\right)u_e(\theta)\left(\bar{R}_{co_2}-\tau_{co_2}\alpha e(\theta)\right).$$

Hence, relative change of the numerator of (40) (also accounting for (39), is given by:

$$\frac{E[\mathcal{G}''\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta)\left(\bar{R}_{co_2}-\tau_{co_2}\alpha e(\theta)\right)-\mathcal{G}'\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta)\alpha\tau_{co_2}|\theta\geq\theta^*]}{E[\mathcal{G}'\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta,\mathcal{P})|\theta\geq\theta^*]}.$$

Analogously, for the denominator, it is

$$\frac{E[\mathcal{G}''\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta)\left(\bar{R}_{co_2}-\tau_{co_2}\alpha e(\theta)\right)-\mathcal{G}'\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta)\alpha\tau_{co_2}]}{E[\mathcal{G}'\left(V(\theta,\mathcal{P})\right)\omega(\theta)u_e(\theta,\mathcal{P})]}.$$

We now define the expectation operator

$$E_{\mathcal{G}'\times\omega\times u_e\times h}\Big[\frac{\mathcal{G}''}{\mathcal{G}'}\left(\bar{R}_{co_2}-\tau_{co_2}\alpha e(\theta)\right)-\alpha\tau_{co_2}|\theta\geq x\Big],$$

where the subscript captures the weighting in the expectation. The second part in the expectation operator is independent of θ^*. We therefore define

$$\xi(x)=E_{\mathcal{G}'\times\omega\times u_e}\Big[\Psi(\theta)|\theta\geq x\Big],$$

where

$$\Psi(\theta)=\frac{\mathcal{G}''}{\mathcal{G}'}\left(\bar{R}_{co_2}-\tau_{co_2}\alpha e(\theta)\right).$$

A necessary condition for the distributional gains term to decrease due to the carbon dividend is

$$\xi(\theta^*)>\xi(\underline{\theta}), \tag{41}$$

since $\xi(\theta^*)$ captures the relative change of the numerator and $\xi(\underline{\theta})$ the relative change of the denominator. There exists a threshold $\tilde{\theta}$ such that $\Psi(\theta)>(<)0$ if $\theta<(>)\tilde{\theta}$. Intuitively, $\tilde{\theta}$ is the productivity type for whom the carbon dividend equals exactly the carbon tax burden. For $\theta^*>\tilde{\theta}$, condition (41) always holds. For $\theta^*<\tilde{\theta}$, a sufficient condition for (41) to hold is $\Psi'(\theta)>0$. To check for this condition, rewrite $\Psi(\theta)$ as

$$\Psi(\theta)=\underbrace{-\frac{\mathcal{G}''(V(\theta))V(\theta)}{\mathcal{G}'(V(\theta))}}_{\equiv CRUIA(\theta)}\times\left(-\frac{1}{V(\theta)}\left(\bar{R}_{co_2}-\tau_{co_2}\alpha e(\theta)\right)\right).$$

Differentiating yields

$$\begin{aligned}\Psi'(\theta) = &- CRUIA'(\theta)\frac{1}{V(\theta)}\left(\bar{R}_{co_2} - \tau_{co_2}\alpha e(\theta)\right) \\ &+ CRUIA(\theta)\frac{1}{V(\theta)^2}V'(\theta)\left(\bar{R}_{co_2} - \tau_{co_2}\alpha e(\theta)\right) \\ &+ CRUIA(\theta)\frac{1}{V(\theta)}\tau_{co_2}\alpha e'(\theta).\end{aligned}$$

By assumption, the aversion to relative utility inequality is not increasing, $CRUIA'(\theta) \leq 0$. Hence, the first line is non-negative. The second line is positive for all $\theta < \theta^*$, and the third line is also positive. Together, this establishes that $\Psi'(\theta) > 0$ thus that (41) holds.

We have therefore shown that the distributional gains term is lower under the carbon dividend than under the status quo. As explained in the main text, since the efficiency cost term remains unchanged compared to the status quo policies, the government optimally reduces redistribution compared to a carbon dividend.

B Quantitative Appendix

B.1 Data

We use three primary data sources: (i) detailed income and consumption expenditure data from the *Einkommens- und Verbrauchsstichprobe* (EVS), provided by the German Statistical Office (Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) 2018b, Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) 2018a), (ii) environmentally extended multi-regional supply-use and input-output tables from EXIOBASE v3.6 and (iii) German administrative income tax data from the *Lohn- und Einkommenssteuerstatistik* (LESt), provided by the German Federal Statistical Office (Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) 2018c). All datasets refer to the year of 2018, except for the LESt, which refers to 2017. We limit our sample to the working-age population, defined as tax units in which the main earner is between 20 and 60 years old.

EVS. The EVS provides quarterly household-level data on consumption expenditures, gross labor income, and socioeconomic characteristics. We convert all monetary variables to an annual level by multiplying the quarterly values by four. The unit of observation is the household. To ensure comparability with income tax data from the LESt, we convert the data from the household level to the tax unit level. Specifically, we split non-married couples into separate tax units by allocating household consumption expenditures according to each person's share of household labor income. This leaves us with a sample of around 26 000 tax units.

Consumption expenditures are reported in purchaser's prices and classified at the five-digit COICOP system. For tractability, we aggregate expenditures to the four-digit COICOP level by summing across all corresponding five-digit subcategories, resulting in 110 different products and services.[26] Table 5 documents the aggregation and provides the relevant variables in the EVS.

We define gross labor income as the sum of income from employment and self-employment. Accordingly, we assign each tax unit to the corresponding percentile of the gross labor income distribution obtained from the LESt. Transfer payments include unemployment benefits (*Ar-*

[26]Detailed expenditure data for food (c01.1), non-alcoholic (c01.2) and alcoholic beverages (c02.1) is not available in the main EVS file (gf5). Therefore, we rely on supplementary data from the gf4 file, which contains detailed recording booklet for food, beverages and tobacco products, covering roughly 20% of the tax units included in the main EVS file (gf5). We compute the expenditure share of each product within the COICOP groups food (c01.1), non-alcoholic beverages (c01.2) and alcoholic beverages (c02.1) at the tax unit level. These shares are then averaged across tax units within net income bins, using survey weights. Finally, we impute consumption expenditures in the gf5 file by multiplying total expenditure in each COICOP group (EVS variables EF242, EF243, and EF244) by the corresponding average expenditure share.

beitslosengeld II; Sozialgeld nach Sozialgesetzbuch II), employment promotion transfers, housing allowance, social assistance and child supplement.[27]

To address the well-known issue of underreporting in household survey data, we scale consumption expenditures in the EVS by a constant factor such that the population-weighted aggregate consumption in the EVS aligns with national accounts data from German Statistical Office (2024a). Similarly, we adjust transfer receipts by applying transfer specific scaling factors to ensure consistency with administrative aggregates. Specifically, unemployment benefits are scaled to match total reported spending from Bundesagentur für Arbeit (2018), housing allowances to aggregates from German Statistical Office (2024b), social assistance to aggregates from German Statistical Office (2025a), and child supplements to aggregates from Familienkasse Direktion SR1 (2018).[28]

EXIOBASE. EXIOBASE contains monetary supply-use and input-output tables with physical extensions, including detailed emission accounts. Specifically, it provides air emission data for 27 pollutants, disaggregated by industry and final demand. EXIOBASE encompasses 44 countries and five rest of the world countries as well as 200 products. The tables are provided in the ISIC classification and in basic prices (Stadler et al. 2018). We use EXIOBASE to compute emission intensities of different goods and services. See Miller and Blair (2009) for a detailed overview of input-output analysis.

LESt. The LESt comprises a 10% cross-sectional random sample of German taxpayers based on administrative income tax records. We use data on annual gross labor income to calibrate the income distribution. Note that the unit of observation is the tax unit. This implies that if a married couple files taxes separately, we cannot identify them as belonging to the same household in our data. In 2018, around 8.1% of married couples filed taxes separately. Labor income of a tax unit is defined as the sum of income from employment and self-employment. Tax payments are calculated as the sum of the assessed income tax (*festzusetzende Einkommenssteuer*), the solidarity surcharge (*Solidaritätszuschlag*) and the church tax.[29] Our sample includes around 2.78 million tax units.

Code	Product	EVS Variables	Brown/Green	Carbon Footprint
		Durables		
c03.1.1	Clothing materials	EF247	Brown	0.4637 kgCO2
c03.1.2	Garments	EF248 - EF250	Brown	0.3975 kgCO2
c03.1.3	Other articles of clothing and clothing accessories	EF251	Brown	0.2961 kgCO2

[27] Gross labor income is computed as the sum of the following EVS variables: EF109U - EF115U, EF118U - EF137U, and EF176. Transfer payments are computed as the sum of the following EVS variables: EF149U, EF150U1, EF151U, EF152U, EF154U - EF157U, and EF168U.

[28] We do not scale employment promotion transfer due to lack of aggregate data.

[29] Gross labor income is calculated as the sum of the following LESt variables: C65120, C65100, C65140, C65163 and C65164. Income tax payments are computed as the sum variables C65613, C66104 and C66105. The solidarity surcharge is computed based on the assessed income tax, taking into account the exemption thresholds applicable in 2018.

c03.2.1	Shoes and other footwear	EF254 - EF257	Brown	0.3779 kgCO2
c05.1.1	Furniture and furnishing	EF326, EF327	Brown	0.2451 kgCO2
c05.1.2	Carpets and other floor coverings	EF328, EF329	Brown	0.2772 kgCO2
c05.2.0	Household textiles	EF331, EF332	Brown	0.4637 kgCO2
c05.3.1	Major household appliances whether electric or not	EF333 - EF336	Brown	0.3113 kgCO2
c05.3.2	Small electric household appliances	EF337	Green	0.1712 kgCO2
c05.4.0	Glassware, tableware and household utensils	EF339, EF340	Brown	0.2723 kgCO2
c05.5.1	Major tools and equipment	EF341, EF342	Green	0.2206 kgCO2
c05.5.2	Small tools and miscellaneous accessories	EF343 - EF345	Brown	0.4800 kgCO2
c07.1.1	Motor cars	EF368, EF369	Green	0.0084 kgCO2
c07.1.2	Motor cycles	EF370	Green	0.0084 kgCO2
c07.1.3	Bicycles	EF371	Brown	0.2683 kgCO2
c07.1.4	Animal drawn vehicles	EF372	Brown	0.3125 kgCO2
c07.2.1	Spare parts and accessories for personal transport equipment	EF374, EF375	Green	0.0745 kgCO2
c08.2.0	Telephone and telefax equipment	EF386	Brown	0.3194 kgCO2
c09.1.1	Equipment for the reception, recording and reproduction of sound and pictures	EF393, EF394	Brown	0.3194 kgCO2
c09.1.2	Photographic and cinematographic equipment and optical instruments	EF395	Brown	0.3194 kgCO2
c09.1.3	Information processing equipment	EF396	Brown	0.3170 kgCO2
c09.1.4	Recording media	EF397	Green	0.1787 kgCO2
c09.2.1	Major durables for outdoor recreation	EF399	Brown	0.3575 kgCO2
c09.3.1	Games, toys and hobbies	EF401	Green	0.0535 kgCO2
c09.3.2	Equipment for sport, camping and open-air recreation	EF402, EF403	Brown	0.2772 kgCO2
c09.3.3	Gardens, plants and flowers	EF404, EF405	Brown	0.2736 kgCO2
c09.3.4	Pets and related products	EF406	Green	0.0003 kgCO2
c09.5.1	Books	EF418	Green	0.1387 kgCO2
c09.5.3	Miscellaneous printed matter	EF421	Green	0.1387 kgCO2
c12.1.2	Electric appliances for personal care	EF438	Green	0.1712 kgCO2
c12.1.3	Other appliances, articles and products for personal care	EF439 - EF441	Green	0.1754 kgCO2
c12.3.1	Jewelry, clocks and watches	EF443, EF444	Brown	0.2644 kgCO2
c12.3.2	Other personal effects	EF445	Green	0.1265 kgCO2
		Electricity		
c04.5.1	Electricity	EF313	Brown	1.4604 kgCO2
		Food		
c01.1.1	Cereals and cereal products	EF58U2 - EF87U2	Green	0.2254 kgCO2
c01.1.2	Live animals, meat and other parts of slaughtered land animals	EF88U2 - EF111U2	Green	0.1969 kgCO2
c01.1.3	Fish and other seafood	EF112U2 - EF125U2	Green	0.0003 kgCO2
c01.1.4	Milk, other diary products and eggs	EF126U2 - EF144U2	Brown	0.2359 kgCO2
c01.1.5	Oils and fats	EF145U2 - EF153U2	Green	0.1976 kgCO2
c01.1.6	Fruits and nuts	EF154U2 - EF177U2	Brown	0.2414 kgCO2
c01.1.7	Vegetables, tubers, plantains, cooking bananas and pulses	EF178U2 - EF208U2	Brown	0.2450 kgCO2
c01.1.8	Sugar, confectionery and desserts	EF209U2 - EF222U2	Brown	0.2296 kgCO2
c01.1.9	Ready-made food and other food products	EF224U2 - EF249U2	Brown	0.2508 kgCO2
c01.2.1	Coffee, tea and cocoa	EF250U2 - EF255U2	Brown	0.2381 kgCO2
c01.2.2	Mineral waters, soft drinks, fruit and vegetable juices	EF256U2 - EF267U2	Green	0.2137 kgCO2
c02.1.1	Spirits	EF268U2 - EF271U2	Brown	0.2520 kgCO2
c02.1.2	Wine	EF272U2 - EF283U2	Brown	0.2520 kgCO2
c02.1.3	Beer	EF284U2 - EF288U2	Brown	0.2520 kgCO2
c02.2.0	Tobacco	EF245	Green	0.1384 kgCO2
		Heating		
c04.5.2	Gas	EF314 - EF317	Brown	1.2191 kgCO2
c04.5.3	Liquid fuels	EF318 - EF320	Green	1.1327 kgCO2
c04.5.4	Solid fuels	EF321	Brown	9.7793 kgCO2
c04.5.5	Heat energy	EF322 - EF324	Green	0.0564 kgCO2
		Housing		
c04.1.1	Actual rentals paid by tenants	EF259 - EF262	Brown	0.1031 kgCO2
c04.1.2	Other actual rentals	EF263 - EF267	Green	0.1021 kgCO2
c04.2.1	Imputed rentals of owner-occupiers	EF268 - EF271	Brown	0.1031 kgCO2
c04.2.2	Other imputed rentals	EF272 - EF274	Green	0.1021 kgCO2
c04.2.3	Rent for garage	EF275, EF276	Brown	0.1045 kgCO2
c04.3.1	Materials for the maintenance and repair of the dwelling	EF277 - EF280	Brown	0.2785 kgCO2
c04.4.1	Operating costs paid by tenants	EF285 - EF287	Brown	1.3233 kgCO2
c04.4.5	Cold operating costs, additional costs	EF297 - EF307	Brown	0.2020 kgCO2
		Other		
c05.6.1	Non-durable household goods	EF346	Brown	0.5872 kgCO2
c06.1.1	Pharmaceutical products	EF349 - EF353	Green	0.0002 kgCO2
c06.1.2	Other medical products	EF354 - EF358	Brown	0.0003 kgCO2
c06.1.3	Therapeutic appliances and equipment	EF359 - EF362	Brown	0.1712 kgCO2
c09.5.4	Stationery and drawing materials	EF422	Brown	0.1240 kgCO2
		Services		
c03.1.4	Cleaning, repair and hire of clothing	EF252, EF253	Brown	0.1240 kgCO2
c03.2.2	Repair and hire of footwear	EF258	Brown	0.1240 kgCO2
c04.3.2	Services for the maintenance and repair of the dwelling	EF281 - EF284	Green	0.0713 kgCO2
c04.4.2	Refuse collection	EF288 - EF290	Brown	0.2564 kgCO2

c04.4.3	Sewage collection	EF291 - EF293	Brown	0.2564 kgCO2
c04.4.4	Other services relating to the dwelling	EF294 - EF296	Brown	0.2020 kgCO2
c05.1.3	Repair of furniture, furnishings and floor coverings	EF330	Brown	0.1240 kgCO2
c05.3.3	Repair of household appliances	EF338	Brown	0.1240 kgCO2
c05.6.2	Domestic services and household services	EF347, EF348	Brown	0.1240 kgCO2
c06.2.1	Medical services	EF363	Brown	0.1173 kgCO2
c06.2.2	Dental services	EF364	Brown	0.1173 kgCO2
c06.2.3	Paramedical services	EF365, EF366	Brown	0.1225 kgCO2
c06.3.0	Hospital services	EF367	Brown	0.1173 kgCO2
c07.2.3	Maintenance and repair of personal transport equipment	EF377	Green	0.0084 kgCO2
c07.2.4	Other services in respect of personal transport equipment	EF378	Brown	0.0967 kgCO2
c08.1.0	Postal services	EF385	Green	0.0706 kgCO2
c08.3.0	Telephone and telefax services	EF387 - EF392	Green	0.0706 kgCO2
c09.1.5	Repair of audio-visual, photographic and information processing equipment	EF398	Brown	0.1378 kgCO2
c09.2.3	Maintenance and repair of other major durables for recreation and culture	EF400	Brown	0.1233 kgCO2
c09.4.1	Recreational and sporting services	EF407 - EF409	Brown	0.0893 kgCO2
c09.4.2	Cultural services	EF410 - EF416	Brown	0.0893 kgCO2
c09.4.3	Games of chance	EF417	Brown	0.1091 kgCO2
c09.5.2	Newspapers and periodicals	EF419, EF420	Brown	0.1284 kgCO2
c10.1.0	Pre-primary and primary education	EF425 - EF427	Green	0.0535 kgCO2
c10.2.0	Secondary education	EF428	Green	0.0535 kgCO2
c10.5.0	Education not definable by level	EF429, EF430	Green	0.0535 kgCO2
c11.1.1	Restaurants, cafés and the like	EF431	Green	0.0729 kgCO2
c11.1.2	Canteens	EF432	Green	0.0729 kgCO2
c11.2.0	Accommodation services	EF433	Green	0.0729 kgCO2
c12.1.1	Hairdressing salons and personal grooming establishments	EF434 - EF437	Brown	0.1265 kgCO2
c12.4.0	Social protection (private)	EF446 - EF449	Brown	0.1093 kgCO2
c12.5.1	Insurance connected with transport	EF460	Green	0.0557 kgCO2
c12.5.2	Other insurance	EF458, EF459, EF461 - EF467	Brown	0.0884 kgCO2
c12.6.2	Other financial services	EF451	Brown	0.1021 kgCO2
c12.7.0	Other services	EF452	Brown	0.1032 kgCO2
		Transport		
c07.2.2	Fuels and lubricants for personal transport equipment	EF376	Brown	1.9436 kgCO2
c07.3.1	Passenger transport by railway	EF379	Green	0.1352 kgCO2
c07.3.2	Passenger transport by road	EF380	Green	0.1165 kgCO2
c07.3.3	Passenger transport by air	EF381	Green	0.8989 kgCO2
c07.3.4	Passenger transport by sea and inland waterways	EF382	Green	1.9249 kgCO2
c07.3.5	Combined passenger transport	EF383	Green	0.2030 kgCO2
c07.3.6	Other purchased transport services	EF384	Green	0.2030 kgCO2
		Vacation		
c09.6.1	Package holidays (Germany)	EF423	Green	0.1468 kgCO2
c09.6.2	Package holidays (foreign country)	EF424	Brown	0.4756 kgCO2

Table 5: Classification of Products

Note: The COICOP groups c01.1 (food), c01.2 (non-alcoholic beverages), and c02.1 (alcoholic beverages) are based on item-level expenditure data from the EVS recording booklet for food, beverages and tobacco products (gf4). All remaining COICOP groups are based on data from the EVS main file (gf5). Carbon footprint are expressed in kgCO2 per euro of expenditure. Within each product category, products are classified as either 'brown' or 'green' based on whether their carbon footprint per euro of expenditure is above or below the expenditure-weighted median for that category. The methodology for computing carbon footprints is described in Appendix B.2.

B.2 Computation of Emission Intensities and Carbon Footprints

The vector of emission intensities is defined as the sum of indirect and direct emissions:

$$\mathbf{f} = \mathbf{f_{ind}} + \mathbf{f_{dir}},$$

where $\mathbf{f} \in \mathbb{R}^{n\times 1}$ contains the carbon footprint in kgCO2 per euro of expenditure for each of the n COICOP categories. The vector $\mathbf{f_{ind}} \in \mathbb{R}^{n\times 1}$ is the carbon footprint arising from indirect emissions and $\mathbf{f_{dir}} \in \mathbb{R}^{n\times 1}$ is the carbon footprint arising from direct emissions.

To compute indirect emission intensities, we must first construct a crosswalk that links expenditure data from the EVS to the input-output structure of EXIOBASE. Following Steen-Olsen, Wood, and Hertwich (2016), Oswald, Owen, and Steinberger (2020) and Hardadi, Buchholz, and Pauliuk (2021), this process involves three steps: (i) mapping COICOP categories to ISIC classification, (ii) converting expenditures from purchaser's prices to basic prices, and (iii) allocating consumption expenditures across all EXIOBASE regions. These steps are described in Sections B.2.1, B.2.2 and B.2.3, respectively. Based on that, Section B.2.4 outlines the computation of indirect emission intensities, and Section B.2.5 details the computation of the direct emission intensities and Section B.2.6 then combines these intensities to compute the carbon footprint of each tax unit observed in the EVS and presents summary statistics.

Throughout our analysis, we focus on carbon dioxide (CO_2) emissions. Other greenhouse gases, such as methane (CH_4), nitrous oxide (N_2O) and sulfur hexafluoride (SF_6), are excluded because CO_2 is the dominant contributor to greenhouse gas emissions in Germany, accounting for 88% of total emissions in 2018 (Umweltbundesamt 2025).[30] Calculating emission intensities in CO_2-equivalents (based on 100-year global warming potentials) does not affect our quantitative results, as the relative composition of CO_2 and other greenhouse gases is approximately constant along the income distribution.

B.2.1 COICOP to ISIC classification

To construct a mapping from COICOP categories to ISIC sectors, we start with an initial correspondence matrix that specifies a binary mapping between COICOP categories and ISIC sectors. We then apply the RAS method to iteratively adjust this matrix so that row and column totals align with final demand in each ISIC sector and COICOP category, respectively. This approach results in a many-to-many mapping, where consumption expenditures in each COICOP category can be distributed across multiple ISIC sectors.

RAS Method. For the initial correspondence matrix, we use an adapted version of the binary mapping proposed by Tisserant (2016) and Hardadi, Buchholz, and Pauliuk (2021). Formally, let $\mathbf{A}_0 \in \mathbb{R}^{m\times n}$ denote the initial correspondence matrix, where m is the number of ISIC sectors and n is the number of COICOP categories. The elements of $\mathbf{A}_0$, denoted as $a_{ij} \in \{0, 1\}$, take the value 1 if COICOP category j is assigned to ISIC sector i (based on product similarities), and 0 otherwise. The RAS method adjusts $\mathbf{A}_0$ iteratively as follows:

[30] See also Statistisches Bundesamt (2025).

1. **Row Adjustment**: The rows of the correspondence matrix $\mathbf{A}^{k-1}$ at iteration k are scaled to match the final demand in each ISIC sector from EXIOBASE, given by the column vector $r \in \mathbb{R}^{m \times 1}$.[31] The updated correspondence matrix is given by

$$\mathbf{A}^{k} = \hat{\mathbf{r}}^{k} \mathbf{A}^{k-1},$$

where the diagonal matrix of row scaling factors $\hat{\mathbf{r}}^{k} \in \mathbb{R}^{m \times m}$ is defined as

$$\hat{\mathbf{r}}^{k} = \text{diag}\left(\left[\frac{r_i}{\sum_{j=1}^{n} \mathbf{A}_{ij}^{k-1}} \right]_{i=1}^{m} \right)$$

with r_i being the i-th element of vector r and $\sum_{j=1}^{n} \mathbf{A}_{ij}^{k-1}$ being the row sum of matrix $\mathbf{A}^{k-1}$.[32] Note that at the first iteration ($k = 1$), the matrix $\mathbf{A}^{k-1}$ is given by the initial correspondence matrix A_0.

2. **Column Adjustment**: The columns of the correspondence matrix $\mathbf{A}^{k-1}$ at iteration k are scaled to match the total expenditures in each COICOP category from EVS, given by the column vector $s \in \mathbb{R}^{n \times 1}$[33]. The updated correspondence matrix is given by

$$\mathbf{A}^{k+1} = \mathbf{A}^{k} \hat{\mathbf{s}}^{k} = \hat{\mathbf{r}}^{k} \mathbf{A}^{k-1} \hat{\mathbf{s}}^{k},$$

where the diagonal matrix of column scaling factors $\hat{\mathbf{s}}^{k} \in \mathbb{R}^{n \times n}$ is defined as

$$\hat{\mathbf{s}}^{k} = \text{diag}\left(\left[\frac{s_j}{\sum_{i=1}^{m} \mathbf{A}_{ij}^{k}} \right]_{j=1}^{n} \right)$$

with s_j being the j-th element of vector s and $\sum_{i=1}^{m} \mathbf{A}_{ij}^{k}$ being the column sum of the matrix $\mathbf{A}^{k}$.[34]

These two steps are repeated iteratively until convergence, ensuring that the row sums of $\mathbf{A}^{k+1}$ closely match r and the column sums of $\mathbf{A}^{k+1}$ closely match s.[35] Once the algorithm converges, we define the resulting correspondence matrix as $\tilde{\mathbf{A}} \in \mathbf{R}^{m \times n}$, which assigns consumption expenditures in COICOP category j to ISIC sectors i.

[31] Final demand in EXIOBASE is reported in basic prices. To transform it into purchaser prices, we add taxes, transport and trade margins to the final demand in basic prices using the Supply and Use Tables from EXIOBASE.

[32] Note that this requires $\sum_{j=1}^{n} \mathbf{A}_{ij}^{k-1} \neq 0 \; \forall \; i$.

[33] Final demand in each COICOP category corresponds to total household consumption expenditure, aggregated using survey weights.

[34] Note that this requires that $\sum_{i=1}^{m} \mathbf{A}_{ij}^{k} \neq 0 \; \forall \; j$.

[35] Note that due to sparse rows in the initial correspondence matrix, the row sums may not exactly match the final demand vector from EXIOBASE. On average, the deviation between the row sum of the correspondence matrix and r is approximately 17%. This issue is common when applying the RAS method to construct a mapping between different classification systems of final demand, particularly when some categories have limited overlap.

Normalization of Correspondence Matrix. Next, we normalize the correspondence matrix $\tilde{\mathbf{A}}$ by dividing each element by the total expenditures of the corresponding COICOP category. Formally, the normalized correspondence matrix is given by

$$\tilde{\mathbf{A}}^{\mathbf{norm}} = \tilde{\mathbf{A}} \oslash (\mathbf{1}^{\mathsf{T}}\tilde{\mathbf{A}}),$$

where $\oslash$ denotes element-wise division, $\mathbf{1} \in \mathbb{R}^{mx1}$ is a column vector of ones and thus, $\mathbf{1}^{\mathsf{T}}\tilde{\mathbf{A}} \in \mathbb{R}^{1\times n}$ is a row vector containing the column sums of $\tilde{\mathbf{A}}$. This normalization ensures that the sum of each column in $\tilde{\mathbf{A}}^{\mathbf{norm}}$ equals 1. Conceptually, the normalized correspondence matrix allocates one euro of purchaser price expenditure in each COICOP category across ISIC sectors.

B.2.2 Purchaser's to Basic Prices

We convert the normalized correspondence matrix $\tilde{\mathbf{A}}^{\mathbf{norm}}$ from purchaser prices into basic prices by deducting taxes and trade margins.[36] Formally, the conversion is given by

$$\tilde{\mathbf{A}}^{\mathbf{norm}}_{\mathbf{bp}} = ((1 - \boldsymbol{\tau} \otimes \mathbf{1}) \odot (1 - \boldsymbol{\mu} \otimes \mathbf{1})) \odot \tilde{\mathbf{A}}^{\mathbf{norm}},$$

where $\tilde{\mathbf{A}}^{\mathbf{norm}}_{\mathbf{bp}} \in \mathbb{R}^{m\times n}$ is the normalized correspondence matrix in basic prices. The tax rate vector $\boldsymbol{\tau} \in \mathbb{R}^{m\times 1}$ and the trade margin rate vector $\boldsymbol{\mu} \in \mathbb{R}^{m\times 1}$ are derived from EXIOBASE.[37] The operator $\otimes$ denotes the Kronecker product, the operator $\odot$ denotes the Hadamard product and $\mathbf{1} \in \mathbb{R}^{1\times n}$ is a row vector of ones. We abstract from deducting transport margins as transport margins are zero for all ISIC sectors in 2018.

Finally, we reallocate trade margins from producing sectors to the wholesale trade service sector and the retail trade service sector. This ensures that trade margins originally embedded in the purchaser prices are correctly attributed to the sectors providing those services. Formally, the basic price entries in the normalized correspondence matrix for the wholesale and retail trade service sector $i \in \mathcal{S}$ are computed as

$$\tilde{a}^{norm}_{bp,ij} = \gamma_i \sum_{p\in\mathcal{P}} \mu_p(1-\tau_p)\tilde{a}^{norm}_{pj} \quad \forall\, j \in \{1, ..., n\},$$

[36]The transformation follows the standard price structure in input-output models, in which basic prices are first augmented by trade margins and subsequently by product taxes to arrive at purchaser prices. Accordingly, we first remove product taxes and then deduct trade margins.

[37]To compute tax rates as well as transport and margin rates for each ISIC sector, we use information from the Supply and Use (SUT) Tables in EXIOBASE. The tax rate for each ISIC sector is defined as the ratio of product taxes to household final demand in purchaser prices. Let $\mathbf{T} \in \mathbb{R}^{m\times 1}$ denote the vector of product taxes and $\mathbf{Y}^{\mathbf{SUT}}_{\mathbf{pp}} \in \mathbb{R}^{m\times 1}$ the vector of household final demand in purchaser prices. Then, the tax rate vector $\boldsymbol{\tau} \in \mathbb{R}^{m\times 1}$ is defined as $\boldsymbol{\tau} = \mathbf{T} \oslash \mathbf{Y}^{\mathbf{SUT}}_{\mathbf{pp}}$. The margin rate is calculated by dividing trade margins by final demand in purchaser prices net of taxes. Let $\mathbf{M}^{\mathbf{trade}} \in \mathbb{R}^{m\times 1}$ denote the vector of trade margins. Then, the margin rate vector $\boldsymbol{\mu} \in \mathbb{R}^{m\times 1}$ is defined as $\boldsymbol{\mu} = \mathbf{M}^{\mathbf{trade}} \oslash (\mathbf{Y}^{\mathbf{SUT}}_{\mathbf{pp}} - \mathbf{T})$. To avoid division by zero, we set $\tau_i = 0$ ($\mu_i = 0$) for all ISIC sectors i for which $Y^{SUT}_{pp,i} = 0$ ($Y^{SUT}_{pp,i} - T_i = 0$).

where $\mathcal{P} \subset \{1, ..., m\}$ denotes the set of product-producing sectors. The share of trade margins allocated to trade sector i is defined as $\gamma_i = \frac{m_i}{\sum_{s \in \mathcal{S}} m_s} \; \forall \; i \in \mathcal{S}$, where $\mathcal{S} \subset \{1, ..., m\}$ denotes the set of trade service sectors and m_i is the i-th element of the trade margins vector $\mathbf{M^{trade}} \in \mathbb{R}^{m \times 1}$ obtained from EXIOBASE. By construction, $\sum_{s \in \mathcal{S}} \gamma_s = 1$.

B.2.3 Allocation across EXIOBASE Regions

In the final step, we disaggregate the normalized correspondence matrix in basic prices $\tilde{\mathbf{A}}_{\mathbf{bp}}^{\mathbf{norm}}$ across the 49 EXIOBASE regions. While household consumption expenditures in the EVS are reported by product category at the national level, it does not provide information on the region in which the production of the consumed goods and services occurs. Thus, we allocate household consumption expenditures across EXIOBASE regions in proportion to each region's share in national final demand for the corresponding ISIC product, as reported in EXIOBASE. Formally, the allocation is given by

$$\mathbf{M} = \text{diag}\left(\text{vec}\left(\mathbf{S}^\mathsf{T}\right)\right) \cdot \left(\mathbf{1} \otimes \tilde{\mathbf{A}}_{\mathbf{bp}}^{\mathbf{norm}}\right),$$

where $\mathbf{1} \in \mathbb{R}^{k \times 1}$ is a column vector of ones and $\mathbf{S} \in \mathbb{R}^{k \times m}$ contains the regional shares of national final demand with $k \in \{1, ..., q\}$ indexing the EXIOBASE regions. The regional share matrix $\mathbf{S}$ is computed as

$$\mathbf{S} = \mathbf{Y_{DE}} \oslash (\mathbf{1}^\mathsf{T} \mathbf{Y_{DE}}),$$

where $\mathbf{Y_{DE}} \in \mathbb{R}^{k \times m}$ denotes final demand of German households by EXIOBASE region q and ISIC sector i, obtained from EXIOBASE.[38] $\mathbf{1} \in \mathbb{R}^{k \times 1}$ is a column vector of ones, so that $\mathbf{1}^\mathsf{T} \mathbf{Y_{DE}}$ yields a row vector of total national final demand for each ISIC sector. Finally, the matrix $\mathbf{M} \in \mathbb{R}^{k \cdot m \times n}$ contains a mapping from one Euro of consumption expenditures in COICOP category j to the corresponding ISIC sectors i and EXIOBASE regions q.

B.2.4 Indirect Carbon Emission Intensity

To transform one euro of household consumption expenditures into indirect carbon emissions, we use the environmentally extended multi-regional input-output framework provided by EXIOBASE. The vector of indirect emission intensities is given by

$$\mathbf{f_{ind}} = \mathbf{E} \cdot \mathbf{L} \cdot \mathbf{M},$$

where $\mathbf{E} \in \mathbb{R}^{1 \times k \cdot m}$ is a row vector of emission factors (in kgCO2 per monetary unit of output), $\mathbf{L} \in \mathbb{R}^{k \cdot m \times k \cdot m}$ is the multi-regional Leontief inverse matrix, and $\mathbf{M} \in \mathbb{R}^{k \cdot m \times n}$ maps consumption expenditures from COICOP categories to ISIC sectors and EXIOBASE regions.

[38] To avoid division by zero, we set the entire column $S_{\cdot,i} = 0$ for all ISIC sectors i for which the total national demand $\sum_k Y_{k,i}$is zero.

The emission factors $\mathbf{E}$ are derived from EXIOBASE's satellite accounts as

$$\mathbf{E} = \mathbf{F} \oslash \mathbf{x}^{\mathsf{T}},$$

where $\mathbf{x} \in \mathbb{R}^{k \cdot m \times 1}$ denotes total economic output (in million euros) by sector and region and $\mathbf{F} \in \mathbb{R}^{1 \times k \cdot m}$ is a row vector containing the total amount of carbon dioxide emissions emitted by each sector-region combination.[39]

The Leontief inverse matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ describes the total input required from each sector-region combination to produce one unit of final demand in another sector-region combination. Here, $\mathbf{I} \in \mathbb{R}^{k \cdot m \times k \cdot m}$ is the identity matrix and $\mathbf{A} \in \mathbb{R}^{k \cdot m \times k \cdot m}$ is the matrix of technical input coefficients obtained from EXIOBASE.[40]

Intuitively, the product $\mathbf{E} \cdot \mathbf{L}$ yields total carbon dioxide emissions per unit of final demand in each sector and region. Multiplying this with $\mathbf{M}$ gives the vector of indirect emission intensities per euro of expenditure in purchaser prices for each COICOP category.

B.2.5 Direct Carbon Emission Intensity

To compute direct carbon emission intensities, we apportion total national direct carbon emissions across COICOP categories (Steen-Olsen, Wood, and Hertwich 2016, Hardadi, Buchholz, and Pauliuk 2021). Let $\mathbf{f_{dir}}(\theta) \in \mathbb{R}^{n \times 1}$ denote the vector of direct emission intensities for each COICOP category. Formally, direct carbon emission intensities are computed as

$$\mathbf{f_{dir}} = \left(\boldsymbol{\epsilon} \cdot \frac{F_{dir}}{N} \right) \oslash \overline{\mathbf{Y}}_{\mathbf{pp}}^{\mathbf{COICOP}},$$

where F_{dir} denotes total national direct carbon emissions from German households, N is the number of tax units in 2018, and the vector $\boldsymbol{\epsilon} \in \mathbb{R}^{n \times 1}$ contains the shares of total direct carbon emissions attributable to each COICOP category. F_{dir} is taken from EXIOBASE and $\boldsymbol{\epsilon}$ are derived from national emission data (German Statistical Office 2023) and summarized in Table 7. The term $\boldsymbol{\epsilon} \cdot \frac{F_{dir}}{N}$ thus represents the average direct carbon emissions per tax unit for each COICOP category. Dividing this by average consumption expenditures $\overline{\mathbf{Y}}_{\mathbf{pp}}^{\mathbf{COICOP}} \in \mathbb{R}^{n \times 1}$ yields the emission intensity from direct emissions.

We only attribute direct emissions to products that generate emissions through their consumption at the household level. These include gas (c04.5.2), liquid fuels (c04.5.3), solid fuels (c04.5.4) as well as fuels and lubricants for personal transport equipment (c07.2.2). As a result, all entries of $\boldsymbol{\epsilon}$ and $\mathbf{f_{dir}}$ are zero for all other COICOP categories.

[39]Note that we aggregate all carbon dioxide related stressor categories provided in the satellite accounts in EXIOBASE to obtain $\mathbf{F}$. We set $E_r = 0$ for all sector-region combinations r for which $x_r < 1$ to avoid division

Energy Source	Direct CO_2 Emissions (kt)	Share (ϵ)
Gas (c04.5.2)	**60 993**	**0.26**
Liquefied Petroleum Gas (LPG)	1 141	
Natural Gas	59 744	
Biomethane	108	
Other Gases	47	
Liquid Fuels (c04.5.3)	**33 292**	**0.14**
Heating Oil	33 292	
Solid Fuels (c04.5.4)	**30 687**	**0.13**
Coal	2 329	
Solid Biomass (wood, pellets, etc.)	28,295	
Fuels for Personal Transport Equipment (c07.2.2)	**107 276**	**0.46**
Gasoline	56 795	
Diesel Fuel	45 445	
Biodiesel	2 527	
Bioethanol	2 509	
Total Direct CO_2 Emissions (F_{dir})	232 231	**1.00**

Table 7: Direct Carbon Emissions of Private Households by Energy Source (2018)

Notes: Data on direct carbon emissions of private households for housing and transport is taken from table `85531-07` from the German Statistical Office (2023). Direct CO_2 emissions are measured in kilotons. All values refer to the year of 2018.

B.2.6 Computation of Carbon Footprint

The carbon footprint of a tax unit observed in the EVS is obtained by multiplying the vector of emission intensities with the corresponding vector of expenditures. Formally,

$$f(\theta) = \mathbf{f}^{\top}\mathbf{Y}_{\mathbf{pp}}^{\mathbf{COICOP}}(\theta),$$

where $\mathbf{Y}_{\mathbf{pp}}^{\mathbf{COICOP}} \in \mathbb{R}^{n\times 1}$ denotes the vector of consumption expenditures across all n COICOP categories, expressed in purchaser's prices.

Descriptive Statistics. Based on our calculations, the mean carbon footprint of a tax unit in Germany is 10.6 tCO2, corresponding to 6.15 tCO2 per capita. Aggregating across all tax units, implies total annual emissions of 458 420 ktCO2, of which roughly 28% are attributable to direct emissions.[41] Our estimates are of the same order of magnitude as the official estimates reported by German Statistical Office (2025b), although our results indicate a slightly lower level of total carbon emissions.

by zero or unrealistically high emission intensities resulting from very low levels of economic output (Hardadi, Buchholz, and Pauliuk 2021).

[40] Denote by $\mathbf{y}$ the vector of final demand. In the basic input-output model, the economy is described by $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y}$. Rearranging gives $(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{y}$, and, provided that $(\mathbf{I} - \mathbf{A})$ is invertible, $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \mathbf{L}\mathbf{y}$.

[41] All descriptive statistics are computed using the EVS survey weights, ensuring representativeness of the estimates at the national level.

B.3 Calibration of Utility Function

B.3.1 Income Effects

Figure 6 illustrates the income effect $\eta(\theta)$ implied by the calibrated model along the gross labor income distribution. The shape and magnitude of the income effect are broadly consistent with the empirical estimates reported by Golosov, Graber, Mogstad, and Novgorodsky (2023).

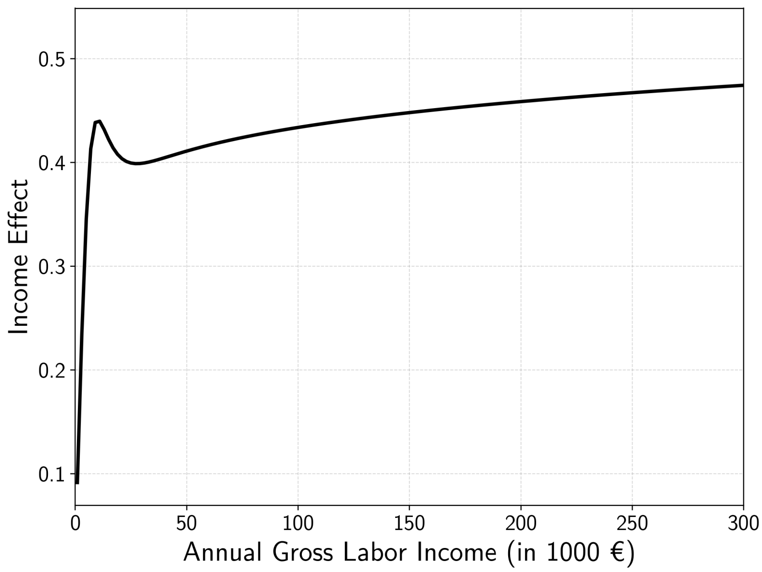


Figure 6: Income Effects

Notes: The figure shows the calibrated income effect $\eta(\theta)$. The x-axis shows gross labor income y of a tax unit.

B.3.2 Internally Calibrated Parameters

Parameters of Lower-Tier Utility Function. We calibrate the CES weights Ω_{ij} for the 'brown' and 'green' varieties within each product i based on expenditure shares from the EVS. Let s_{ib} denote the expenditure share on the brown variety, and let p_{ib} and p_{ig} denote the respective prices of the brown and green varieties. The CES structure of the lower-tier utility function implies

$$s_{ib} = \frac{p_{ib}^{1-\sigma_i}\Omega_{ib}^{\sigma_i}}{\sum_{j\in\{b,g\}} p_{ij}^{1-\sigma_i}\Omega_{ij}^{\sigma_i}}.$$

Without loss of generalization, we normalize the CES weight on the green variety to one $\Omega_{ig} = 1\ \forall i$. Solving for Ω_{ib} then yields

$$\Omega_{ib} = \left(\left(\frac{1}{s_{ib}} - 1\right)\left(\frac{p_{ib}}{p_{ig}}\right)^{\sigma_i - 1}\right)^{-1/\sigma_i} \qquad \forall i. \tag{42}$$

Using the average expenditure share on the brown variety across income quartiles from the EVS, and normalizing prices to unity, i.e. $p_{ij} = 1 \ \forall i, j$, we compute Ω_{ib} for each product using (42).

Parameters of Upper-Tier Utility Function. We calibrate the expenditure elasticities of demand $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_I)$ and the taste shifter $\boldsymbol{\Omega} = (\Omega_1, \ldots, \Omega_I)$ by minimizing the squared distance between the model-implied and empirical differences in expenditure shares between top and bottom quartile of the gross labor income distribution, subject to matching the average expenditure shares observed in the data. Formally, the parameters $(\boldsymbol{\epsilon}, \boldsymbol{\Omega})$ are the solution to the optimization problem

$$\min_{\boldsymbol{\epsilon},\boldsymbol{\Omega}} \left(\boldsymbol{\Gamma}^{model} - \boldsymbol{\Gamma}^{data}\right)^\top \left(\boldsymbol{\Gamma}^{model} - \boldsymbol{\Gamma}^{data}\right) \quad \text{s.t.} \quad \bar{\mathbf{s}}^{model}(\boldsymbol{\epsilon}, \boldsymbol{\Omega}) = \bar{\mathbf{s}}^{data},$$

where $\boldsymbol{\Gamma}^{model} = \boldsymbol{s}_{Q4}^{model} - \boldsymbol{s}_{Q1}^{model}$ is the vector of model implied differences in expenditure shares between the top and bottom quartile of the gross labor income distribution, and $\boldsymbol{\Gamma}^{data} = \boldsymbol{s}_{Q4}^{data} - \boldsymbol{s}_{Q1}^{data}$ contains the corresponding empirical moments from the EVS. The vectors $\boldsymbol{s}_{Qq} = (s_{1q}, s_{2q}, \ldots, s_{Iq})^\top$ denote expenditure shares on each of the $I = 9$ products for households in income quartile $q \in \{1, 4\}$, and $\bar{\mathbf{s}} = (\bar{s}_1, \ldots \ \bar{s}_I)$ denotes the average expenditure share across all households for each product $i \in \{1, \ldots, I\}$.

B.4 Tax-Transfer System

We estimate the parameters of the tax and transfer function separately using nonlinear least squares. The estimation sample is restricted to tax units with positive labor income. To ensure a good fit across the entire income distribution, we group observations into percentiles of the gross labor income distribution prior to estimation. The parameters are then estimated using these percentile-level data.

Tax Function. We estimate the parameters of the tax function (θ, τ) using the following regression equation:

$$y_i^{atax} = \left(1 - \exp\left[\log(\lambda)(y_i/\bar{y})^{-2\tau}\right]\right) y_i + u_i,$$

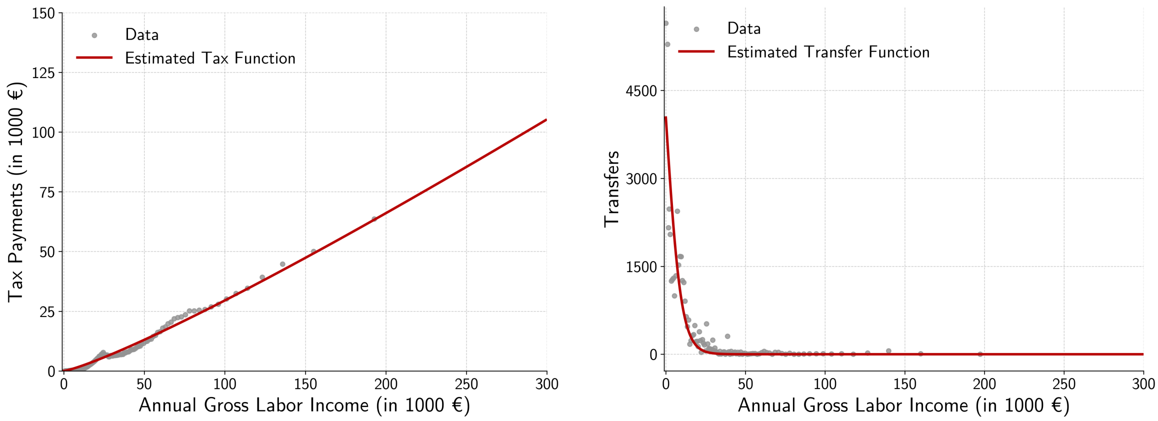
where y_i^{atax} denotes after-tax labor income, y_i is gross labor income of tax unit i, $\bar{y}$ is average gross labor income, and u_i is the error term.

Transfer Function. We estimate the parameters of the transfer function (m, ζ) using the following regression equation

$$\mathcal{T}_{0_i} = \hat{m}\bar{y}\frac{2\exp\left[-\zeta\frac{y_i}{\bar{y}}\right]}{1 + \exp\left[-\zeta\frac{y_i}{\bar{y}}\right]} + u_i,$$

where $\mathcal{T}_{0_i}$ is the received transfer of tax unit i.

Estimation Fit. Figure 7 illustrates the estimation fit by comparing estimated taxes and transfers with their data counterparts. Overall, the tax-transfer function provides a good fit of the observed tax-transfer schedule in Germany.



(a) Tax Function

(b) Transfer Function

Figure 7: Estimated Tax-Transfer Function

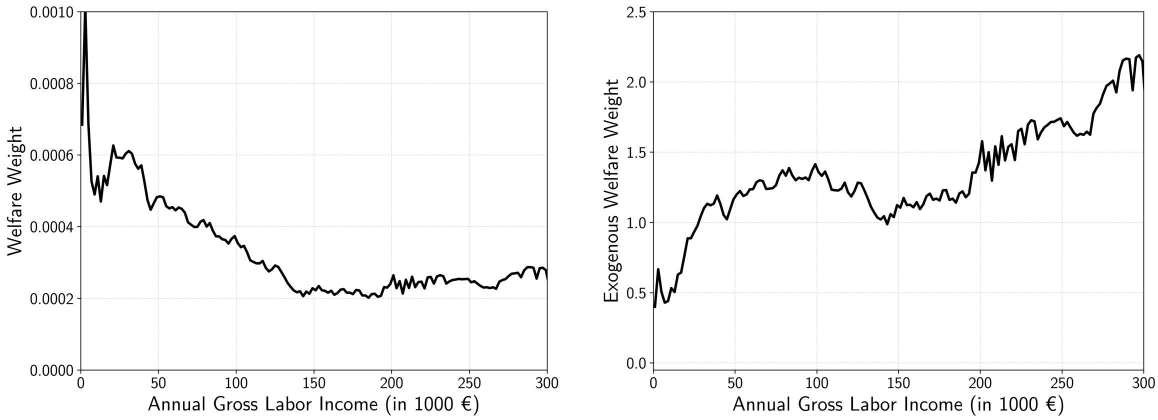
Notes: The left panel shows the estimated tax function and the right panel the estimated transfer function (red solid lines). The gray dots correspond to the observed average tax payments and transfer payments, respectively. Observations are grouped into percentiles of the gross labor income distribution. The x-axis shows gross labor income y of a tax unit.

B.5 Income Distribution

Kernel Density Estimation. To obtain a smooth estimate of the income distribution from the LESt, we apply a standard kernel density estimation over log labor income using a Gaussian kernel and survey weights. The sample is restricted to tax units with positive labor income. We use an evenly spaced log income grid with 5000 nodes ranging from €0.8 to €10 470 000. The bandwidth is selected according to Silverman's rule of thumb.

B.6 Welfare Weights

Figure 8(b) shows the calibrated social marginal utility and Figure 8(a) shows the Pareto weights along the gross labor income distribution.



(a) Social Marginal Utility (b) Exogenous Welfare Weights

Figure 8: Calibrated Welfare Weights

Notes: The left panel shows the social marginal utility $g(\theta, \mathcal{P}_{sq})$ and the right panel shows the Pareto weights $\omega(\theta)$ as a function of gross labor income y of a tax unit.

B.7 Robustness

B.7.1 Optimal Carbon Rebate

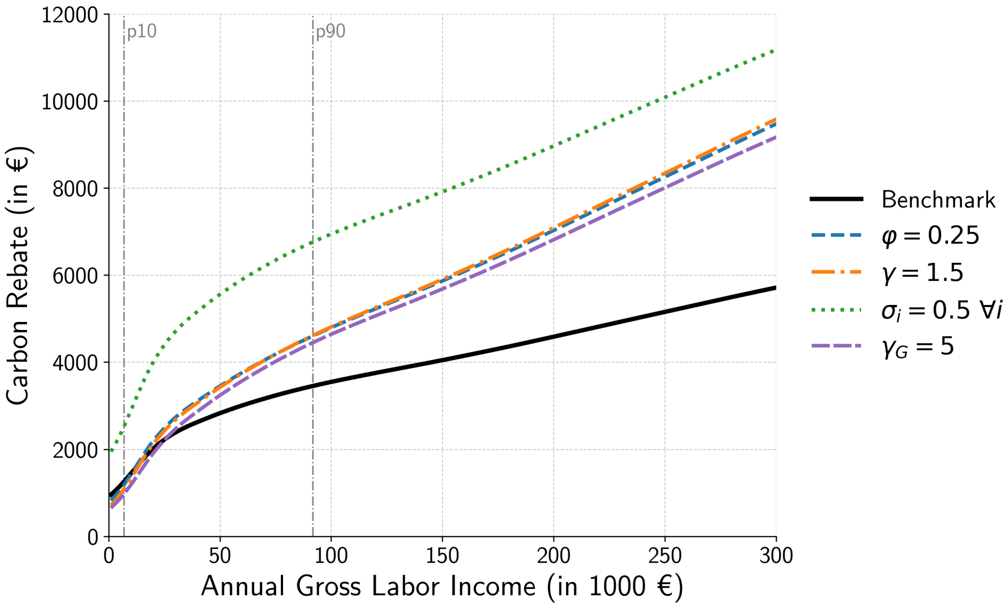


Figure 9: Optimal Carbon Rebate (Different Specifications)

Notes: The figure shows the optimal carbon rebate as a function of gross household income for five alternative parameter specifications. The case *pro-rich substitution* refers to a scenario in which the elasticity of substitution between the brown and green varieties differs across consumption categories, allowing richer households to substitute more easily from brown to green varieties. The gray vertical dashed-dotted lines correspond to the 10th and 90th percentile of the annual gross labor income distribution.

B.7.2 Redistributive Implications

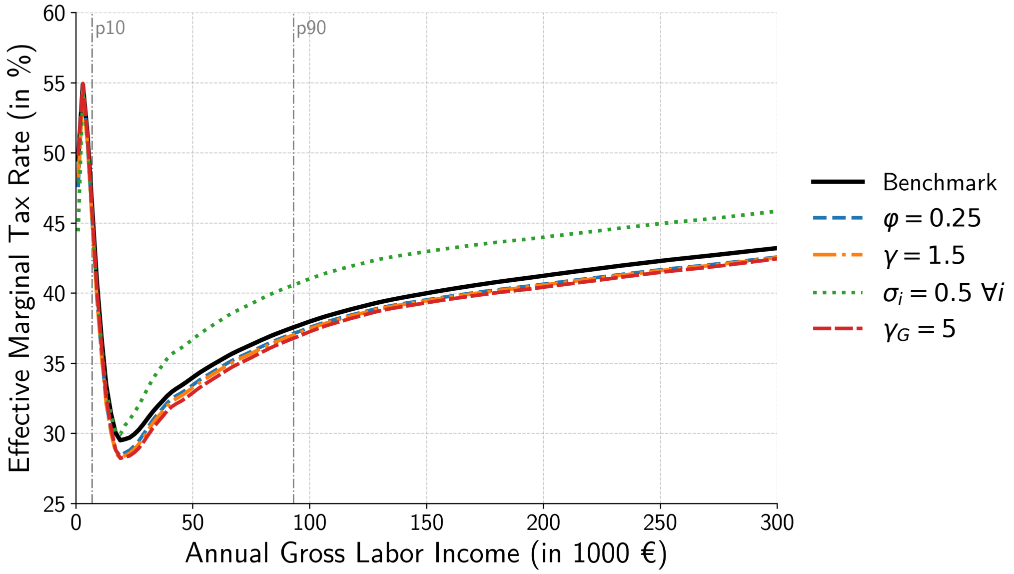


Figure 10: Effective Marginal Tax Rate (Different Specifications)

Notes: The figure shows the effective marginal tax rate τ_{eff} as a function of gross labor income for five alternative parameter specifications. The case *pro-rich substitution* refers to a scenario in which the elasticity of substitution between the brown and green varieties differs across consumption categories, allowing richer households to substitute more easily from brown to green varieties.

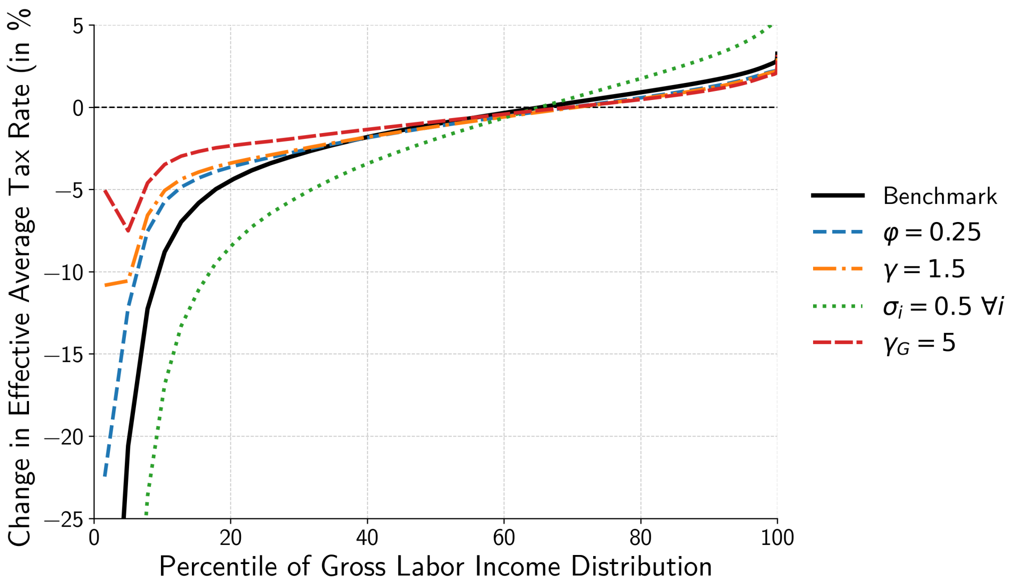


Figure 11: Net Burden of Carbon Policy (Different Specifications)

Notes: The figure shows the change in the effective average tax rate $\bar{\tau}_{eff}$ across percentiles of the gross labor income distribution, as defined in (15), for five alternative parameter specifications. The case *pro-rich substitution* refers to a scenario in which the elasticity of substitution between the brown and green varieties differs across consumption categories, allowing richer households to substitute more easily from brown to green varieties.