Pareto-improving reforms of tax deductions[*]

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Abstract

We analyze Pareto-efficient tax deduction rules for work-related expenses. Pareto efficiency dictates a strict rule for marginal deductions along the income distribution. An immediate implication is a recipe for designing Pareto-improving reforms. We apply our theory and simulate a Pareto-improving reform that introduces deductions for non-care household services (housekeeping, gardening, laundry) in the United States. The reform combines marginal deduction rates for household services between 55% and 85% with a slight increase in marginal tax rates.

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1 Introduction

Tax codes for personal taxation contain a large number of tax deductions, sometimes also called tax expenditures or tax breaks. Many of those deductions concern work-related expenses. For example, the U.S. tax code promotes investments in individual earnings ability, e.g. expenses for education and health. Expenses for services that provide child care or long-term care for elderly parents, and thereby free up time for market work, can also be deducted in various ways. Similar provisions exist in many other OECD countries.

Such deduction possibilities are often considered to be inequitable or regressive because high income earners make use of them to a larger extent.[1] In this paper, we argue that such concerns are likely to be misplaced because the design of deduction rules can be separated from distributional concerns. We provide a recipe to redesign a given tax system (jointly changing marginal tax rates and deduction rules) in a Pareto-improving way: tax revenue is increased and nobody is made worse-off.

Our analysis extends the literature on commodity taxation and the literature on Pareto-efficient income taxation. A well-known result on commodity taxation is that complements to work should be subsidized by the tax system (Corlett and Hague, 1953; Atkinson and Stiglitz, 1976; Christiansen, 1984). In particular, services that save time should be taxed at relatively low rates (Kleven, 2004). Although these results generally motivate tax deductions for work-related goods and services, several open questions remain: *How large should the deductions be? How should they vary across heterogeneous individuals? How can policy reforms be designed in a Pareto-improving way?* In particular, although the optimal tax literature contains a large number of simulations focusing on optimal tax rates, there are hardly any attempts to quantify optimal deductions (or subsidies) for specific expenses—the cases of child care (e.g., Bastani et al., 2020; Ho and Pavoni, 2020) and charitable giving (Saez, 2004) are rare exceptions. Secondly, our paper extends the literature on Pareto-efficient income taxation (see Werning (2007) for the static case and Kapicka (2020) for a dynamic extension) by introducing tax deductions as a second policy dimension.[2] The two-dimensional policy space makes the test

[1] The Congressional Budget Office of the U.S. addresses this issue in their economic and budget outlook: "Tax expenditures are distributed unevenly across the income scale."(Congressional Budget Office, 2016, p.103). They also devoted an extra report to it (Congressional Budget Office, 2013). This report has been taken up also by the press, see e.g. Washington Post (2013).

[2] Laroque (2005) and Kaplow (2006) study the Pareto-efficient taxation of income and commodities when the goods are separable from work. In their framework, the famous uniform commodity taxation result applies and,

of Pareto efficiency more powerful and imposes a strict rule for marginal deductions at each income level (irrespective of distributional properties). This result differs markedly from the pure income tax setting without work-related goods and without deduction rules: Werning (2007) shows that for each tax schedule in the classical Mirrlees setup, there exists a skill distribution such that the tax schedule is Pareto efficient.

Our main findings are as follows. First, by solving a resource maximization problem holding individual utilities at given baseline levels (e.g., the utility levels implied by current policies), we provide a characterization of Pareto-efficient deduction rules for a general model of work-related goods (Proposition 1 and Corollary 1). Our formula shows how Pareto-efficient deduction rules are determined by preference nonseparabilites between work-related goods, skills and labor supply.[3] In its general version, the formula suggests that Pareto-efficient deduction rules may be relatively complex and hints that simple deduction rules (e.g., zero or full deductibility of work-related expenses within a continuous income range) are possibly Pareto inefficient.

Second, we specialize the model of work-related goods and consider time-saving services that substitute the taxpayer's engagement in non-market work (housekeeping, gardening, etc.). For those services, we derive an easily-testable necessary condition for Pareto efficiency: expenses on time-saving services should be positively *but less than fully* deductible from taxable income (Proposition 2). An immediate policy implication is that the lack of tax deductions for household services (for households without children or other dependents in need of care) in the U.S. tax code is Pareto inefficient. Another implication is that tax credits that exceed a full deductibility are also Pareto inefficient. In particular, the Swedish form of tax credits amounting to fifty percent of household service expenses are not well designed for at least the lower part of the income distribution.[4]

Third, to quantitatively explore the cross-sectional properties of Pareto-efficient policies and to assess the magnitude of the inefficiency in the U.S. tax code, we simulate a Pareto-improving reform that introduces deductions for non-care household services (housekeeping, gardening,

hence, the second policy dimension is redundant (Atkinson and Stiglitz, 1976). Koehne (2018) shows that the uniform taxation result fails in dynamic environments with durable goods.

[3]The formula is based on our structural model and cannot easily be expressed in a sufficient-statistics form. Relying on sufficient statistics is generally difficult when the policy space is multi-dimensional—even for a simple case with income taxes and work-related subsidies that are separable from each other, the optimality conditions will involve at least two elasticities that lack empirical evidence. We discuss this issue in more detail in Appendix A.5.

[4]See Section 4.2.1 for further details on current deduction rules in the U.S., Sweden and other countries.

laundry) for single, prime-age households. Our results show that all households spend less on household services and supply less labor in the status quo than in a Pareto-efficient outcome. We outline a Pareto-improving reform that increases the marginal deduction rates for household services from zero in the status quo to 55% for low expense levels and up to 85% for high expense levels. At the same time, marginal tax rates are shifted up slightly, in particular for lower incomes. Importantly, the reform constitutes a Pareto improvement despite its seemingly regressive nature—nobody is worse off and tax revenue is increased. The limited magnitude of the gains reflects the fact that annual expenses for non-care household services are generally rather low and range from 100 to 1000 Dollars along the income distribution, as we document with data from the Consumer Expenditure Survey.

We deliberately abstract from child care in our quantitative application. Child care services are obviously time-saving in the sense that they reduce the parents' engagement in non-market work. We therefore conjecture that child care expenses will also be imperfectly deductible in a Pareto-efficient tax system. However, child care is a more complicated issue than standard housekeeping services due to human capital consequences of care (for children and parents) and, relatedly, due to a more obvious quality dimension of care compared with housekeeping services. Those and other properties of child care are addressed for example by Bastani et al. (2020) and Ho and Pavoni (2020). A major difference between our contribution and those papers is that we theoretically and quantitatively study Pareto-improving policies. Moreover, we evaluate our theory for household services other than care and we propose tax deductions as a general implementation device for constrained-efficient allocations when the Atkinson-Stiglitz theorem fails.

The paper is also related to the recent literature on human capital subsidies (e.g., Findeisen and Sachs, 2016; Stantcheva, 2017). We especially relate to work by Kapicka (2020) that studies the evolution of labor wedges across time in a learning-by-doing framework. Moreover, the paper is related to the literature on Pareto-efficient income taxation. Recent papers by Scheuer (2014), Lorenz and Sachs (2016), Hendren (2017), Scheuer and Werning (2017), and Bierbrauer et al. (2020) abstract from tax deductions and study reforms of the marginal tax rates on income.

Saez (2004) studies deductions for charitable giving in a model with a contribution good, linear taxes and subsidies, and social welfare maximization. Based on numerical simulations, his paper suggests that subsidy rates on charitable giving should typically lie below the earnings tax rate. Our results propose time-saving services as another policy-relevant class of expenses that should be imperfectly deductible from taxable income.

2 Model

Individuals supply labor and choose how to allocate their income between two consumption goods. One of these goods is nonseparable with labor and represents work-related consumption. Examples include services that free up time for market work (e.g., services for housekeeping, gardening, etc.), job-related equipment, apparel, books and home offices. More broadly, the work-related good may also capture a health investment. The work-related good is feasible for a tax deduction. The second consumption good represents general consumption and is separable from labor.

2.1 Preferences

Individuals are heterogeneous in their skill $n \in \mathcal{N} := [n_0, n_1] \subset \mathbb{R}_{++}$. The distribution of skill types in the economy is defined by a smooth probability density $f : \mathcal{N} \to \mathbb{R}_{++}$ with full support. Preferences are described by a continuously differentiable function $u : \mathbb{R}^4_+ \to \mathbb{R}$. Utility $u(c, d, y; n)$ is strictly increasing in general consumption c and work-related consumption d, strictly decreasing in output (pre-tax labor income) y, and concave with respect to (c,d,y).

Throughout the paper, we assume that utility is additively separable between general consumption and output:

$$u(c, d, y; n) = w(c, d) + v(d, y; n), \tag{1}$$

where w and v are continuously differentiable, concave in (c, d), and $w(c, d)$ is strictly increasing in c and weakly increasing in d, whereas $v(d, y; n)$ is strictly increasing in d and strictly decreasing in y. This functional form draws a clear distinction between general consumption c and work-related consumption d based on the separability of the former from the disutility of work. The main purpose of the functional form is to facilitate the exposition and interpretation of the theoretical results. Yet, the general approach of the paper does not hinge on this assumption.

Although we focus on goods that are positively related to work, our concept of work-related goods is broad and covers any good that is nonseparable with the disutility of work. In particular, any good that is substitutable or complementary with leisure will be classified as work-related in our terminology.[5] Moreover, our formulation includes deterministic models of human capital formation.[6]

2.2 Tax system

Individuals face a nonlinear labor income tax schedule $\mathcal{T} : \mathbb{R} \to \mathbb{R}$ with a deduction rule $\mathcal{D} : \mathbb{R}^2_+ \to \mathbb{R}$ for work-related expenses. The deduction rule $\mathcal{D}(y, d)$ may be nonlinear and nonseparable between labor income y and work-related consumption d. The individual tax payment is given by $\mathcal{T}(y - \mathcal{D}(y, d))$, where $y - \mathcal{D}(y, d)$ represents the taxable income of the individual. We call the pair $(\mathcal{T}, \mathcal{D})$ a tax system.

Note that our specification of tax systems covers the entire universe of nonlinear, nonseparable tax functions. For any function $\hat{\mathcal{T}}(y, d)$, there trivially exists an equivalent tax system with a deduction rule for work-related expenses that yields the same tax payments. For example, let $\mathcal{T}$ be the identity function and set

$$\mathcal{D}(y, d) := y - \hat{\mathcal{T}}(y, d).$$

Then, by definition,

$$\hat{\mathcal{T}}(y, d) = \mathcal{T}(y - \mathcal{D}(y, d)) \quad \forall (y, d).$$

2.3 Individual problem and wedges

Individuals maximize their welfare given the tax system. They solve the following problem:

$$\max_{(c,d,y) \in \mathbb{R}^3_+} u(c, d, y; n) \text{ s.t. } c + d = y - \mathcal{T}(y - \mathcal{D}(y, d)) \tag{2}$$

[5] For leisure complements, Pareto-efficient tax deduction rules will typically be negative.
[6] See Sections 3.3.3 and 3.3.4 for more details.

The first-order conditions imply

$$
\begin{aligned}
-\frac{u_y}{u_c} &= 1 - (1 - \mathcal{D}_y)\, \mathcal{T}' \\
\frac{u_d}{u_c} &= 1 - \mathcal{D}_d\, \mathcal{T}'.
\end{aligned}
$$

As these conditions show, the labor supply decision and the expenditure on the work-related good are both influenced by the marginal tax rate at the individual's taxable income as well as the marginal deduction rule. These two policy instruments jointly determine the respective wedges. First, there is the *labor wedge* τ_y given by

$$
\tau_y := 1 + \frac{u_y}{u_c} = (1 - \mathcal{D}_y)\, \mathcal{T}'. \tag{3}
$$

The labor wedge measures the gap between the marginal rate of transformation and the marginal rate of substitution between pre-tax income and consumption. As the right-hand side of Eq. (3) shows, the gap is induced by the tax system in the following way: an extra dollar of pre-tax income increases the individual's taxable income by $(1 - \mathcal{D}_y)$ dollars; therefore, the tax bill grows by $(1 - \mathcal{D}_y)\mathcal{T}'$ dollars. If the deduction rule depends on labor income, the labor wedge does not simply equal the marginal tax rate. For example, if higher income allows for more deductions ($\mathcal{D}_y > 0$), the distortion on labor supply is lower than the mere accounting for the marginal tax rate would suggest.

The second relevant wedge in our environment is the *expenditure wedge* τ_d. We define this wedge as the gap between the marginal rate of transformation between work-related goods and general consumption and the marginal rate of substitution between the two. Formally, we set

$$
\tau_d := 1 - \frac{u_d}{u_c} = \mathcal{D}_d\, \mathcal{T}'. \tag{4}
$$

This wedge captures the implicit subsidy to work-related goods relative to general consumption. Note that an extra dollar of work-related spending reduces the individual's taxable income by $\mathcal{D}_d$ dollars and diminishes the tax bill by $\mathcal{D}_d\mathcal{T}'$ dollars. By contrast, an extra dollar spent on general consumption is not deductible.

There are two natural benchmarks for the work-related expenditure wedge. If work-related spending is not deductible from taxable income ($\mathcal{D} = 0$), we have $\tau_d = 0$. By contrast, if work-related goods can be paid with pre-tax income ($\mathcal{D} = d$), i.e, if these expenses are fully deductible, we have $\tau_d = \tau_y$.

More generally, the difference between the two wedges captures the overall distortion to work-related spending induced by the tax system. We define the *net expenditure wedge* as

$$\bar{\tau}_d := \tau_y - \tau_d = (1 - \mathcal{D}_y - \mathcal{D}_d)\,\mathcal{T}'. \tag{5}$$

A marginal dollar of labor income that is spent on the work-related good increases the agent's tax bill by $\bar{\tau}_d$ dollars. Therefore, a zero net wedge means that the tax system is neutral with respect to work-related spending. For instance, if work-related expenditures are fully deductible irrespective of the level of income, we have $\mathcal{D}_d = 1$, $\mathcal{D}_y = 0$ and therefore $\bar{\tau}_d = 0$. A positive net wedge, by contrast, implies that work-related spending is at most imperfectly deductible.

Because the net wedge is mathematically redundant, we will formulate our theoretical findings mainly in terms of the labor wedge and the expenditure wedge. Yet, the net wedge will be a useful measure of the optimal degree of deductibility in our quantitative evaluation further below.[7]

2.4 Applying the revelation principle

To characterize Pareto-improving reforms of the tax system, we make use of the revelation principle. By the revelation principle, any allocation that can be implemented through a tax system $\mathcal{T}\left(y - \mathcal{D}\left(y, d\right)\right)$, can also be implemented through an incentive-compatible direct mechanism. Formally, an allocation $(c(n), d(n), y(n))_{n\in\mathcal{N}}$ is *incentive compatible* if it satisfies

$$u\left(c\left(n\right), d\left(n\right), y\left(n\right); n\right) \geq u\left(c\left(n'\right), d\left(n'\right), y\left(n'\right); n\right) \quad \forall n, n' \in \mathcal{N}. \tag{6}$$

As usual, individual welfare maximization subject to the tax system establishes an incentive-compatible allocation. A simple application of the taxation principle (Hammond, 1979; Rochet, 1985) implies that the reverse is also true: for any incentive-compatible allocation, there ex-

[7] The concept of the net wedge is similar to the terminology of Bovenberg and Jacobs (2005) measuring whether education is taxed or subsidized on a net basis and to the concept of a net human capital subsidy in Stantcheva (2017).

ists a tax system that implements the allocation. This result also implies that if we find a Pareto-improving and incentive-compatible allocation perturbation, there exists a tax reform that implements this allocation perturbation.[8]

Following common practice in optimal tax theory, we replace the incentive-compatibility constraint by an envelope condition. Specifically, we define the agents' indirect utilities as

$$U(n) = u(c(n), d(n), y(n); n)$$

and replace the incentive-compatibility constraint (6) by the following condition:

$$\dot{U}(n) = v_n(d(n), y(n); n). \tag{7}$$

It is well-known that the envelope condition is necessary for incentive compatibility (e.g., Mirrlees, 1976). The envelope condition is sufficient provided that the second-order condition of utility maximization with respect to the reported type is satisfied.[9]

3 Pareto-improving reforms of tax deduction rules

Consider a given tax system $\mathcal{T}(y - \mathcal{D}(y, d))$ that generates an allocation $(c(n), d(n), y(n))_{n \in \mathcal{N}}$. In this section, we describe how a reform can be constructed in order to achieve a Pareto improvement. We show that such a reform exists unless the wedges implied by the tax system satisfy a continuum of efficiency conditions.

If an allocation is Pareto inefficient, then typically many Pareto-improving reforms exist. We will elaborate on those reforms that maximize tax revenue without altering the levels of individual utilities. Given an incentive-compatible allocation $(c(n), d(n), y(n))_{n \in \mathcal{N}}$ that is induced by the baseline tax system, we seek to find an incentive-compatible allocation $\left(\hat{c}(n), \hat{d}(n), \hat{y}(n)\right)_{n \in \mathcal{N}}$

[8] We present an implementation with an income-independent deduction rule in our quantitative analysis in Section 4.2.5. A general implementation result is derived in Appendix A.2.

[9] As shown by Mirrlees (1976), the envelope condition is sufficient if for all n, n' we have

$$y'(n) v_{yn}\left(d(n), y(n); n'\right) + d'(n) v_{dn}\left(d(n), y(n); n'\right) \geq 0.$$

Because we work with arbitrary baseline allocations, we cannot validate this condition theoretically. In our quantitative application, we verify ex post that the condition is satisfied at the computed allocations.

that maximizes resources

$$\int_{n_0}^{n_1} \left(\hat{y}(n) - \hat{c}(n) - \hat{d}(n) \right) f(n)\, dn$$

subject to:

$$u\left(\hat{c}(n), \hat{d}(n), \hat{y}(n); n \right) = u\left(c(n), d(n), y(n); n \right) \quad \forall n \in \mathcal{N}. \tag{8}$$

Eq. (8) ensures that no individual is worse off after the reform.

3.1 Constructing the reform

For each type n, we change work-related consumption by some (positive or negative) amount ε and we adjust output and general consumption (up or down) such that utility remains unchanged and the envelope condition continues to hold. We seek to reduce the amount of resources needed for the given levels of utility. Formally, for every $n \in \mathcal{N}$, we define the elements $(\hat{c}(n), \hat{d}(n), \hat{y}(n))$ of the perturbed allocation as follows:

$$\begin{aligned} \hat{d}(n) &= d(n) + \varepsilon(n) \\ \hat{c}(n) &= c(n) + \gamma(n) \\ \hat{y}(n) &= y(n) + \delta(n) \end{aligned} \tag{9}$$

subject to the constraints

$$u\left(\hat{c}(n), \hat{d}(n), \hat{y}(n); n \right) = u\left(c(n), d(n), y(n); n \right), \tag{10}$$

$$\frac{du\left(\hat{c}(n), \hat{d}(n), \hat{y}(n); n \right)}{dn} = v_n\left(\hat{d}(n), \hat{y}(n); n \right), \tag{11}$$

where Eq. (10) ensures that no individual is made worse off and Eq. (11) ensures that the reform is incentive compatible.

We make a change of variables and express the consumption perturbation in terms of utility levels $U(n)$:

$$\gamma(n) = w^{-1}\Big(U(n) - v\big(d\,(n) + \varepsilon(n), y\,(n) + \delta\,(n)\,; n\big),\ d\,(n) + \varepsilon(n)\Big) - c\,(n)$$

where w^{-1} denotes the inverse of $w(c, d)$ with respect to its first argument and $U(n)$ represents the utility of an agent with skill n at the original (and perturbed) allocation. Now we obtain an optimal control problem with state variable $U(n)$ and controls $\varepsilon(n)$ and $\delta(n)$:

$$\max_{U(n),\varepsilon(n),\delta(n)} \int_{n_0}^{n_1} \Big[\delta(n) - \varepsilon(n) - w^{-1}\Big(U(n) - v\Big(d(n) + \varepsilon(n), y(n) + \delta(n); n\Big), d(n) + \varepsilon(n)\Big) + c(n)\Big] f(n) dn \tag{12}$$

subject to

$$U(n) = w(c(n), d(n)) + v\,(d(n), y(n); n)$$
$$\dot{U}(n) = v_n\,(d(n) + \varepsilon(n), y(n) + \delta(n); n)\,.$$

3.2 Properties of Pareto-efficient allocations

Problem (12) implies a set of necessary conditions for Pareto efficiency. These conditions not only describe the allocation after a Pareto-improving reform has been implemented. More importantly for our purposes, the conditions are also a test for whether a given allocation is Pareto efficient and, hence, whether or not the reform described in Section 3.1 can yield a Pareto improvement.

By applying the maximum principle for Problem (12), we obtain the following property of Pareto-efficient allocations.[10]

Proposition 1 (Incentive-adjusted no-arbitrage principle) *Consider an interior allocation: $(c, d, y) > 0$. Suppose $v_{ny}(d, y; n) \neq 0$ for all $(d, y; n)$. A necessary condition for Pareto*

[10] For brevity, we abstract from corner solutions and focus on interior allocations. For corner solutions with respect to work-related consumption, the optimality condition (13) takes the form of an inequality whose direction depends on the sign of the cross derivatives of v.

efficiency is that the following condition:

$$\frac{u_c + u_y}{v_{ny}} = \frac{u_c - u_d}{-v_{nd}} \tag{13}$$

holds for all types n with $v_{nd}(d(n), y(n); n) \neq 0$ and that $\tau_d = 0$ holds for all types n with $v_{nd}(d(n), y(n); n) = 0$.

To gain intuition for the Pareto efficiency condition, note that an individual always has two ways to finance a marginal unit of general consumption: the individual can reduce her consumption of the work-related good d by one unit, or work more and increase income y by one unit. These two options change individual utilities by $u_c - u_d$ and $u_c + u_y$, and affect the incentive problem through the envelope condition according to $-v_{nd}$ and v_{ny}, respectively. Thus, Eq. (13) shows that the cost of a marginal unit of general consumption (measured in utility terms relative to incentive costs) must be the same for both ways of financing the marginal unit of consumption. In this sense, Eq. (13) can be interpreted as an *incentive-adjusted no-arbitrage principle.*

Proposition 1 implies that Pareto-efficient deduction rules strongly depend on preference nonseparabilites between work-related goods, skills and labor supply. An alternative way of writing Eq. (13) is to express it in the form of wedges:

$$\tau_d = -\frac{v_{nd}}{v_{ny}} \tau_y. \tag{14}$$

which shows that Pareto efficiency dictates a tight relation between the labor and the work-related good wedge. For the purpose of testing the efficiency of a given tax system $(\mathcal{T}, \mathcal{D})$, we now rephrase Eq. (13) in terms of the tax system.

Corollary 1 *Consider a tax system $(\mathcal{T}, \mathcal{D})$. The allocation implemented by this tax system is Pareto efficient if and only if*

$$\mathcal{D}_d = -\frac{v_{nd}}{v_{ny}} \left(1 - \mathcal{D}_y\right) \tag{15}$$

for all types n where $\mathcal{T}'\left(y - D(y, d)\right) \neq 0$.

Although the right-hand side of Eq. (15) depends on the functional form of the leisure utility function v, this equation already suggests that Pareto-efficient deduction rules may be relatively complex. In particular, unless the ratio of cross derivatives v_{nd}/v_{ny} happens to take a very simple form, standard deduction rules (e.g., zero or full deductibility of work-related expenses within a continuous income range) are unlikely to be Pareto efficient. In such a case, the reform constructed in Section 3.1 will yield a Pareto improvement. In Section 4, we will further elaborate on how such reforms look like for the special case where d represents a time-saving service such as housekeeping. We will also quantitatively apply the insight and construct a Pareto-improving reform for housekeeping-service expenditures in the United States. Before moving to time-saving services, we show in Section 3.3 how our conditions for Pareto efficiency are related to various results in the literature.

3.3 Special cases and extensions

Proposition 1 encompasses several important benchmark results in the optimal taxation literature. Next, we present four well-known cases that have been discussed in the literature. In Section 4 we consider another practically relevant application: services that replace the agent's engagement in non-market work.

3.3.1 Uniform commodity taxation

As shown by Atkinson and Stiglitz (1976), the consumption choice should be undistorted if the preferences are separable between consumption and work. For separable preferences ($v_{nd} = 0$), Proposition 1 indeed implies $\tau_d = 0$.[11] Hence, Proposition 1 shows that uniform commodity taxes are necessary conditions for Pareto efficiency if the preferences are separable between consumption and work, reiterating the findings by Laroque (2005) and Kaplow (2006).

3.3.2 Subsidies to work-complementary goods

If the utility function of leisure takes the common form $v(d, y; n) = \tilde{v}(d, \frac{y}{n})$, we can interpret $l := \frac{y}{n}$ as hours worked and the skill level n as the individual's hourly productivity. In that case, Eq. (14) implies that τ_d has the same sign as the labor wedge if the utility of leisure has

[11] We assume from the start that general consumption is additively separable from the preferences for work. Therefore, strictly speaking, we obtain the Atkinson-Stiglitz result only for the case of additively separable preferences. The uniform taxation result is true more generally whenever the consumption preferences are *weakly* separable from work.

a positive cross derivative with respect to work-related consumption d and hours worked l.[12] In other words, Proposition 1 implies that work-complementary goods are subsidized in any Pareto-efficient allocation. This result is in line with the findings by Christiansen (1984).

3.3.3 Human capital subsidies

Bovenberg and Jacobs (2005) show that education should be subsidized at the exactly same rate as income is taxed. Thus, in our terminology, it would be optimal to have $\tau_d = \tau_y$ when d represents an educational investment. Proposition 1 helps to understand this well-known finding in the theory of optimal education subsidies from a different angle. It also highlights the generality of their finding by showing that marginal education subsidies and marginal income taxes in fact coincide along the entire Pareto frontier in their framework. We obtain their setup if we set

$$u(c, d, y; n) = w(c) - V\left(\frac{y}{n\phi(d)}\right),$$

where $\phi(\cdot)$ is concave and $V(\cdot)$ convex. In that case, we have $-\frac{v_{nd}}{v_{ny}} = \frac{1-\tau_d}{1-\tau_y}$.[13] Hence, by Eq. (14), Pareto efficiency dictates

$$\frac{\tau_d}{1-\tau_d} = \frac{\tau_y}{1-\tau_y},$$

implying that $\tau_d = \tau_y$ holds in any Pareto-efficient allocation. Or, another way to put it, the net wedge is zero, i.e., we have $\bar{\tau}_d = 0$.

3.3.4 Multiple work-related goods and dynamic labor wedges

The no-arbitrage principle of Proposition 1 extends without difficulty to multiple work-related goods. Specifically, consider an environment with a vector $d = (d_1, \ldots, d_K)$ of work-related goods and a utility function of the form $u(c, d, y; n) = w(c, d) + v(d, y; n)$. Analogous to

[12] With $v(d, y; n) = \tilde{v}(d, l)$, we obtain

$$\tau_d = -\frac{v_{nd}}{v_{ny}}\tau_y = \frac{\frac{y\tilde{v}_{dl}}{n^2}}{-\frac{\tilde{v}_l}{n^2} - \frac{y\tilde{v}_{ll}}{n^3}}\tau_y.$$

The denominator of this expression is positive (assuming that the utility of leisure is decreasing and concave in hours worked). The sign of the right-hand side is hence determined by the cross derivative $\tilde{v}_{dl}$.

[13] More precisely, we obtain

$$-\frac{v_{nd}}{v_{ny}} = \frac{V'\frac{y\phi'}{n^2\phi^2} + V''\frac{y^2\phi'}{n^3\phi^3}}{V'\frac{1}{n^2\phi} + V''\frac{y}{n^3\phi^2}} = \frac{y\phi'}{\phi} = \frac{\frac{y\phi'}{n\phi^2}V'}{\frac{1}{n\phi}V'} = \frac{1-\tau_d}{1-\tau_y}.$$

Eq. (4), define the work-related expenditure wedge for good k in this environment as

$$1-\tau_d^k(n) := \frac{u_{d_k}(c(n), d(n), y(n); n)}{u_c(c(n), d(n), y(n); n)}.$$

Then, the approach of Proposition 1 establishes the following necessary condition for Pareto efficiency (assuming $v_{n,d_{k'}} \neq 0$):

$$\frac{\tau_d^k}{\tau_d^{k'}} = \frac{v_{n,d_k}}{v_{n,d_{k'}}} \quad \text{for all } 1 \leq k, k' \leq K. \tag{16}$$

Similar to Eq. (14), this condition states that the wedges should be determined in proportion to the marginal incentive effects of the respective goods.

If the work-related goods do not have a direct consumption value, they become similar to labor supplies in dynamic environments. Therefore, Eq. (16) is related to characterizations of labor wedges across time (in frameworks without uncertainty) as studied by Kapicka (2020). We provide further details in Appendix A.4.

4 An application to time-saving services

We now turn to services that replace the agent's engagement in non-market work. This environment captures several real-world situations. Many individuals hire housekeepers, gardeners or cleaning staff to free up time from domestic chores. Further, individuals pay professionals to care for their children, an ill spouse or elderly relatives.[14] The costs of these services are tax deductible in a number of countries (including Germany, Sweden and Denmark). In this section, we analyze the efficiency of such deductions through the lens of our model.

Throughout this section, we maintain the following assumption.

Assumption 1 (Time-endowment model) *The utility function is given by*

$$u(c, d, y; n) = w(c) + \tilde{v}\left(E(d) - \frac{y}{n}\right) \tag{17}$$

where $\tilde{v}' > 0 > \tilde{v}''$ and $E' > 0 > E''$.

[14] Alternatively, the service may represent a (curative or preventive) health investment that reduces the number of sick days in a given year or delays the worker's retirement.

Under Assumption 1, the worker has a concave utility function $\tilde{v}$ defined over leisure, where leisure is the difference between the endowment of time E (net of non-market work) and hours of labor supply $l = y/n$.

4.1 Imperfect deductibility

Proposition 2 (Imperfect deduction of time-saving services) *Under Assumption 1, a necessary condition for Pareto efficiency is $0 < \tau_d(n) < \tau_y(n)$ whenever the labor wedge of the considered type satisfies $0 < \tau_y(n) < 1$.*

By Proposition 2, Pareto efficiency requires a positive net wedge for time-saving services: $\bar{\tau}_d(n) > 0$, implying that these services should be positively, but imperfectly, deductible at the margin. To understand this result and to elaborate on possible Pareto-improving reforms, we now consider two particularly relevant benchmarks.

Proposition 3 (Introducing a tax deduction for time-saving services) *Suppose that Assumption 1 holds. Starting from a tax system where time-saving services are not deductible and labor wedges are positive, i.e., $0 = \tau_d(n) < \tau_y(n) < 1$, a Pareto-improving reform exists where type-n individuals spend more on time-saving services, work more and consume less.*

The virtue of deductions for time-saving services can be most easily understood in a model version with discrete types. Consider an agent with skill n and a hypothetical shirker with skill $\hat{n} > n$ who mimics the type-n agent. A tax system without deductions of time-saving services can be improved in the following steps. First, we increase time-saving services d by a marginal unit and reduce general consumption c by the same amount. (This step raises the expenditure wedge to a positive level.) Because the margin between time-saving services and general consumption was undistorted at the baseline tax system, the perturbation has no impact on the utility of the truth-telling agent:

$$du = -w' + E'(d)\tilde{v}'\left(E(d) - \frac{y}{n}\right) = 0.$$

The shirking agent, however, consumes more leisure and therefore values a unit of time-saving services relatively less. Her utility thus falls:

$$d\hat{u} = -w' + E'(d)\tilde{v}'\left(E(d) - \frac{y}{\hat{n}}\right) < 0.$$

Hence, the joint change of consumption and time-saving services has relaxed the incentive problem without affecting the utility of the truth-telling agent. In the next step, due to the relaxed incentive-compatibility constraint, it becomes possible to increase consumption and income one-for-one. Given a positive labor wedge, this step will increase the agent's utility. In the final step, we can extract resources to reset the agent's utility to its baseline level.

While Proposition 3 shows that a zero deductibility of time-saving services is inefficient, the following result highlights that the opposite case of a full deductibility is inefficient too. Overall, these results imply that Pareto-efficient tax systems necessarily include imperfect deduction possibilities for time-saving services.

Proposition 4 (Reducing a full tax deduction of time-saving services) *Suppose that Assumption 1 holds. Starting from a tax system where time-saving services are fully deductible and labor wedges are positive, i.e.,* $0 < \tau_d(n) = \tau_y(n) < 1$, *a Pareto-improving reform exists where type-n individuals spend less on time-saving services, work less and consume more.*

To understand Proposition 4, we construct a counterpart to the perturbation described above. Once more, we consider a truth-telling agent with skill n and a hypothetical shirker with skill $\hat{n} > n$. A tax system with a full deduction of time-saving services can be improved in the following steps. First, we perform a joint reduction of income y and time-saving services d by one marginal unit.[15] The utility of the truth-telling agent changes by

$$du = -\left[E' - \frac{1}{n}\right] \cdot \tilde{v}'\left(E(d) - \frac{y}{n}\right),$$

whereas the utility of the shirker changes by

$$d\hat{u} = -\left[E' - \frac{1}{\hat{n}}\right] \cdot \tilde{v}'\left(E(d) - \frac{y}{\hat{n}}\right).$$

[15] Intuitively, time-saving services and reductions of labor supply are substitutable inputs for the production of leisure. The proposed perturbation replaces a unit of time-saving services by an equivalent amount of reduced labor supply, holding constant the level of leisure. As a consequence, the expenditure wedge falls, whereas the labor wedge remains constant.

A full deductibility means that the margin between income and time-saving services is undistorted at the baseline tax system, i.e., we obtain $du = 0$.[16] For the shirking agent, however, the perturbation causes a first-order utility loss, $d\hat{u} < 0$, because reducing the earnings by a dollar does not bring as much extra leisure as for the truth-telling agent. Hence, in a second step, we can increase consumption and income by a small amount without violating the incentive-compatibility constraint. Given a positive labor wedge, this step will increase the agent's utility. Finally, we can extract some resources to reset the agent's utility to its baseline level.

4.2 A quantitative study of tax deductions for housekeeping services

Next, we apply the time-endowment model to assess the potential welfare consequences of introducing tax deductions for housekeeping services in the United States. Unlike some European countries (e.g., Germany, Sweden and Denmark), the current US tax code does not provide general tax breaks to households that hire services for housekeeping, gardening, laundry, et cetera. In this section, we quantitatively evaluate Pareto-improving reforms that stimulate the consumption of such services through tax deductions.

4.2.1 Institutional background

Tax breaks for household service expenditures exist in a number of countries including Denmark, Germany and Sweden. The classification of household services that are eligible for a tax break is similar between those three countries and includes services such as cleaning, gardening and child care.

In Denmark, individuals can deduct up to 6,000 DKK (2018) per person per year for expenses on household services ("servicefradrag"). The deduction reduces the individual's taxable income, implying that the monetary value depends on the personal tax rate. By contrast, Germany and Sweden provide tax credits that directly reduce the income tax liability of the claimant. In Germany, expenses on household services are credited at a rate of 20 percent against the tax liability of the household ("Abzug für haushaltsnahe Dienstleistungen"). The maximum credit per year and household is 4,000 EUR (2018). In Sweden, expenses on household services are credited at a rate of 50 percent and the tax credit is limited to 25,000 SEK (2018) per person per year ("RUT-avdrag"). The marginal rate of the income tax (municipal,

[16] Formally, we have $\tau_d = \tau_y$ if and only if $u_d = -u_y$ if and only if $E'\tilde{v} = -\tilde{v}/n$.

county and national income tax combined) in Sweden has three steps at approximately 32, 52 and 57 percent. Hence, the tax credit rate of 50 percent corresponds to a strong degree of deductibility of household service expenses and for individuals in the lowest tax bracket, in fact, exceeds a full deductibility.

In the United States, the "Child and Dependent Care Credit" provides a tax credit for the care of children (under age 13) and the care of dependents incapable of self-care. Eligible annual expenses are limited to \$3,000 (for one qualifying person) or \$6,000 (for two or more qualifying persons). For taxpayers with incomes greater than \$43,000, the tax credit amounts to 20 percent of the care costs. For taxpayers with lower incomes, the rate can rise to 35 percent. Expenses for other household services qualify only if a part of the service is for the care of qualifying persons. For example, the tax credit is not given for standard cleaning or gardening services. However, the tax credit applies for the employment of a nanny that provides child care and completes ancillary household tasks. As an alternative to the "Child and Dependent Care Credit", some employees have access to a pre-tax dependent care account offered by their employer. The account can be used to deduct up to \$5,000 per year from taxable income to pay for the care of children (under age 13) or dependents incapable of self-care. Once more, household services qualify for the tax break only if the service includes care for a qualifying person.

Summing up, for households without young children or other dependents in need of care, the US tax code does not grant a tax break for expenses on household services. In our quantitative evaluation, we will therefore consider single, prime-age households.

4.2.2 Model specification

We consider a quasi-linear version of the time-endowment model with a utility function of the following form:

$$u\left(c,d,y;n\right)=c+\gamma\frac{\left(1-\left(\frac{\bar{s}-d}{\alpha_1}\right)^{\frac{1}{\alpha_2}}-\frac{y}{n}\right)^{1-\frac{1}{\phi}}}{1-\frac{1}{\phi}}$$

where the parameters $(\alpha_1,\alpha_2,\bar{s},\gamma,\phi)$ are positive. As usual, market labor is given by $l=y/n$ in the above specification.

The utility function is derived from a household production model based on Sandmo (1990) and Kleven et al. (2000). Households produce domestic housekeeping services with a concave technology: $d_n = \alpha_1 l_n^{\alpha_2}$, where l_n represents non-market work and (α_1, α_2) are parameters with $0 < \alpha_1$ and $0 < \alpha_2 < 1$. Household-produced housekeeping services d_n and housekeeping services d obtained from the market are perfect substitutes: $s = d + d_n$. Households have a fixed demand for housekeeping services $s = \bar{s}$ and a fixed time endowment $\bar{E} = 1$ that can be used for market work l, domestic work l_n and leisure.

The household decision problem has a convenient closed-form solution. Given a marginal tax rate $\mathcal{T}'$ and a zero subsidy on housekeeping services, the first-order conditions of the household problem can be solved for housekeeping service expenditure d and labor supply l as follows:

$$d = \bar{s} - \left(\frac{\alpha_1^{\frac{1}{\alpha_2}} \alpha_2}{(1 - \mathcal{T}')\, n} \right)^{\frac{1}{\frac{1}{\alpha_2} - 1}}, \tag{18}$$

$$l = 1 - \left(\frac{\bar{s} - d}{\alpha_1} \right)^{\frac{1}{\alpha_2}} - \left(\frac{\gamma}{(1 - \mathcal{T}')\, n} \right)^{\phi}. \tag{19}$$

4.2.3 Calibration strategy

We calibrate the parameters $(\alpha_1, \alpha_2, \bar{s}, \gamma, \phi)$ by matching a set of moments. Intuitively, the parameters $(\alpha_1, \alpha_2, \bar{s})$ govern the cross sectional expenditure pattern for housekeeping services. The parameter γ determines the time share of labor. Finally, the value of ϕ corresponds to the Frisch elasticity of *leisure* (holding non-market work fixed), which is one-to-one related to the Frisch elasticity of labor supply.[17] Consequently, we target the time share of labor (i.e., market work), the Frisch elasticity of labor supply, and the pattern of housekeeping service expenditures of US households by income. Table 1 summarizes the parameters and data moments of our calibration procedure.

Parameter	Target	Source	Data	Model
ϕ	Mean Frisch elasticity of labor	Chetty et al. (2013)	0.5	0.499
γ	Mean time share of labor	Standard (40h/week)	0.238	0.239
$\alpha_1, \alpha_2, \bar{s}$	Housekeeping service expenditure (by income group)	CEX 2015	See Figure 1	

Table 1: Calibration strategy

[17] More precisely, the Frisch elasticity of labor equals $(1 - l_n - l)\phi/l$, where ϕ is the Frisch elasticity of leisure and $(1 - l_n - l)/l$ is the leisure/labor ratio.

Our measure of housekeeping service expenditure is based on the Consumer Expenditure Survey (CEX) 2015 and consists of housekeeping, gardening and laundry services. Child care and other care services are excluded in our definition. We restrict the data set to single, prime-age households (age 25–54 years) for the sake of comparability across households and to rule out intra-household substitution margins. We group those households into eight income bins (less than 25k, 25-50k, 50-75k, 75-100k, 100-125k, 125-150k, 150-200k, more than 200k) based on their annual employment income in US\$.[18] For each income bin, we compute the means of income and housekeeping service expenditure in the CEX data. Mean housekeeping service expenditure increases with the income group (except for the second and second-to-last income group, where it slightly decreases) and ranges from \$126 to \$937 in our sample (Figure 1).

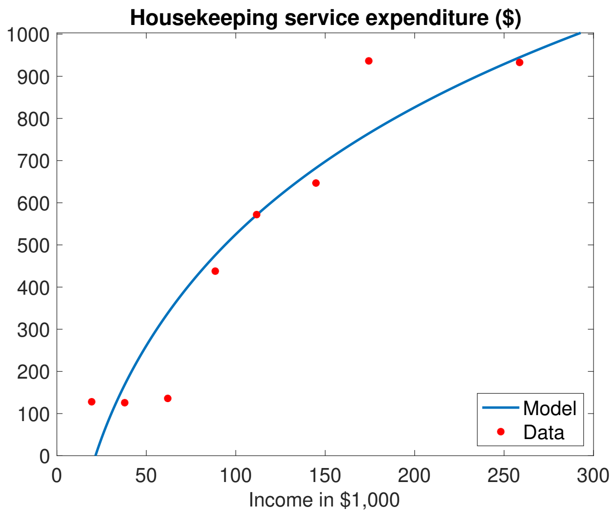


Figure 1: Housekeeping service expenditure in model and CEX data

For the baseline allocation, we assume that the households face marginal tax rates on labor income according to the parametric tax function of Gouveia and Strauss (1994),

$$\mathcal{T}' = \beta \left[1 - \left(\sigma \hat{y}^{\theta} + 1 \right)^{-\frac{1}{\theta} - 1} \right],$$

where income $\hat{y}$ is measured in thousands of US dollars. Guner et al. (2014) provide recent estimates of these parameters for different specifications of the US tax system. We apply their parameters of the specification for unmarried households that includes state and local taxes. Hence, we set $\beta = 0.287$, $\sigma = 0.006$, $\theta = 1.514$. In line with current tax practice in the US, we

[18] We drop households with incomes below the federal poverty level.

set the subsidy on household services to zero at the baseline allocation.

Our calibration proceeds in two steps. First, we construct a vector of skill types n that is consistent with the income groups of the CEX data. By substituting the solution for service expenditure (Eq. 18) into the solution for labor supply (Eq. 19), we obtain

$$\frac{y}{n} = 1 - \left(\frac{\alpha_1 \alpha_2}{(1-\mathcal{T}')\, n} \right)^{\frac{1}{1-\alpha_2}} - \left(\frac{\gamma}{(1-\mathcal{T}')\, n} \right)^{\phi}.$$

For given parameters $(\alpha_1, \alpha_2, \bar{s}, \gamma, \phi)$ and the schedule of marginal tax rates $\mathcal{T}'$, this condition defines a nonlinear equation for skill n for every income level y, similar to the approach by Saez (2001). Based on this equation, we construct a skill vector such that the associated income levels match the incomes of the different groups in the CEX data. In the second step, we use the obtained skill vector, solve the individual decision problem and compare the model prediction with the calibration targets of Table 1. We calibrate the parameters $(\alpha_1, \alpha_2, \bar{s}, \gamma, \phi)$ by minimizing the distance to the calibration targets using standard numerical routines.

As shown by Table 1, our calibration matches the targets for the Frisch elasticity of labor supply and the time share of labor almost perfectly. We also obtain a reasonable fit for the expenditure pattern on housekeeping services (Figure 1).

4.2.4 Characterizing the Pareto-improving allocation reform

After solving for the baseline allocation, we explore a counterfactual Pareto-improving reform based on our theoretical results from Section 3 and maximize the resource gains of the reform. To approximate a continuous support of the skill distribution, we solve the model on a finer skill grid than the one used for the calibration procedure. Given that we start from a baseline allocation without deductions for housekeeping services, Proposition 3 implies that the reform induces households to work more and spend more on household services.

The changes in the allocation variables due to the Pareto-improving reform are illustrated in Figure 2. Evaluated at the median income level of US households in 2015 ($56,516), housekeeping service expenditures increase by $122 under the reform (relative to a baseline level of $304). Income increases by $95 and other consumption falls by roughly $39, resulting in a resource gain of $13 for a median household.

Note that the pattern of the reform is exactly in line with Proposition 3; housekeeping

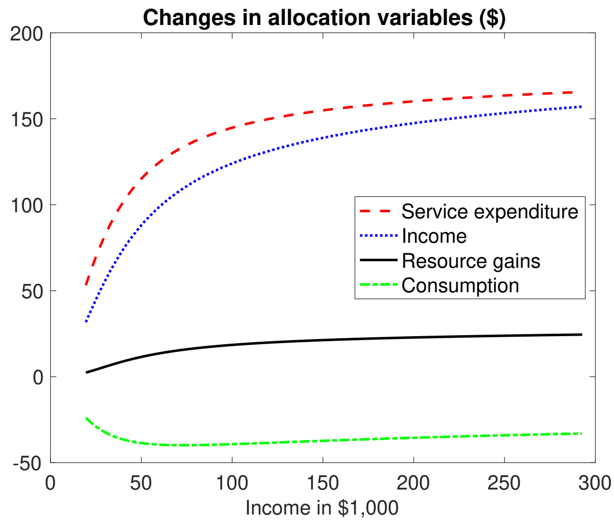
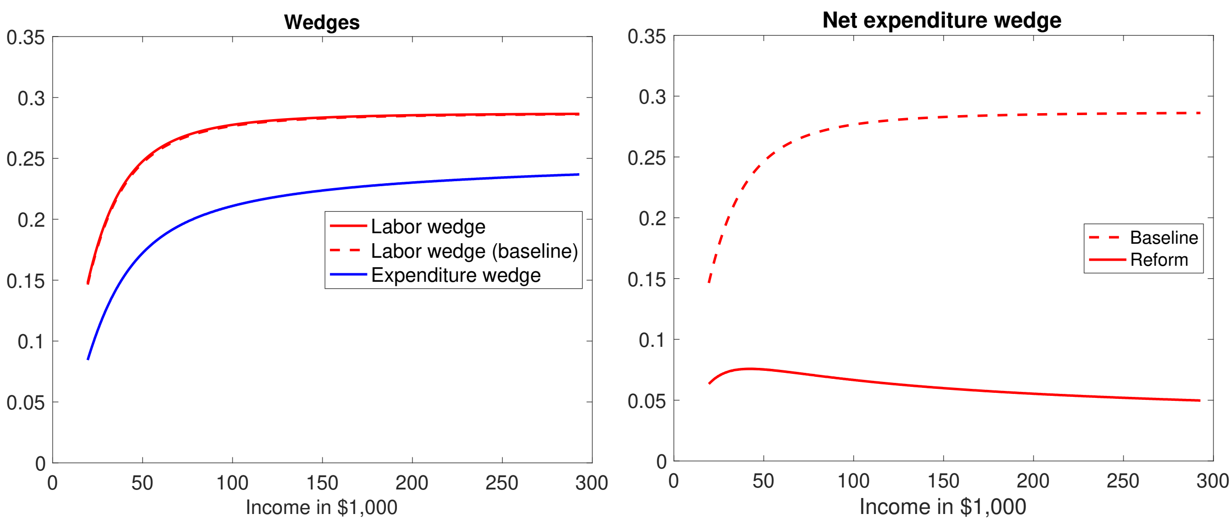


Figure 2: Allocation change after Pareto-improving reform

services and income increase, whereas consumption decreases under the reform. Hence, the general idea of stimulating time-saving services in order to facilitate labor supply in a situation with binding incentive constraints also applies to non-marginal reforms.



(a) Labor and expenditure wedge

(b) Net expenditure wedge for housekeeping services

Figure 3: Wedges after Pareto-improving reform

The wedges for labor and housekeeping service expenditures are shown in Figure 3. Labor wedges remain largely unaffected by the reform, whereas the expenditure wedge increases notably from its baseline level of zero. The net expenditure wedges are relatively uniform across households and lie at approximately 6 percent (Figure 3b). That is, Pareto efficiency requires that expenses on housekeeping services are strongly deductible for all households.

4.2.5 A tax reform implementing the Pareto-improved allocation

Finally, we also characterize a tax reform that implements the Pareto-improved allocation. By the taxation principle, as described in Section 2.4, there exists a tax system implementing any incentive-compatible allocation. Next, we outline a particularly simple implementation of the Pareto-improved allocation and numerically verify its validity.

We consider a deduction rule that depends only on the expenditure level, but not on income:

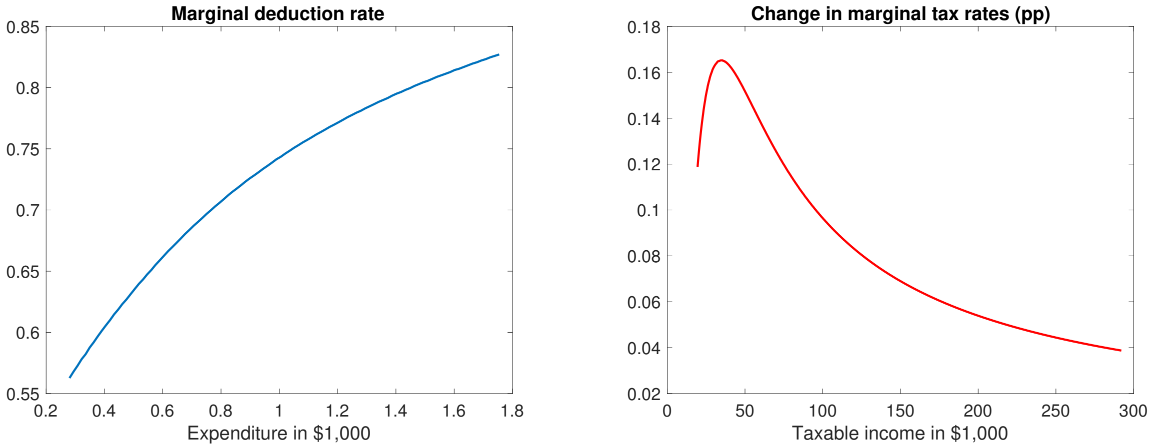
$$\mathcal{D}_y(d, y) = 0 \quad \forall\, d, y.$$

Having made this choice, Eqs. (3) and (4) imply that we have to set the marginal deduction rates and marginal income tax rates in the following way:

$$\mathcal{D}_d(d(n)) = \frac{\tau_d(n)}{\tau_y(n)} \quad \forall\, n,$$
$$\mathcal{T}'(y(n) - \mathcal{D}(d(n))) = \tau_y(n) \quad \forall\, n.$$

By construction, the first-order conditions of the individual choice problem are satisfied given these policy instruments. However, the open question is whether each individual bundle constitutes a global individual optimum. A theoretical analysis of this question is generally rather difficult (e.g., Renes and Zoutman, 2016). Therefore, our approach is to analyze the individual choice problem numerically.

It turns out that our proposed implementation is valid: individuals do choose the same (c, d, y) bundles as in the direct mechanism. Hence, this simple tax reform indeed implements the desired allocation. Figure 4a describes the marginal deduction rates $\mathcal{D}_d$ for the new tax system. They monotonically increase from 55% to 85% across the expenditure range. Figure 4b illustrates the change in marginal tax rates as a function of taxable income $y - \mathcal{D}(d)$. The increase in marginal tax rates is generally modest and never above 0.17 percentage points. This reflects the fact that housekeeping expenditures are generally at a rather low level. To sum up, Figure 4 shows that the Pareto-improving tax reform takes a very simple form here.



(a) Marginal deduction rates for household service expenditures

(b) Change in marginal tax rates

Figure 4: Tax reform implementing the Pareto-improved allocation

5 Conclusion

In this paper, we study the Pareto-efficient design of tax deductions for work-related goods. Our approach also provides guidelines on how inefficient tax systems can be Pareto improved. We quantitatively apply our method to assess the benefits of introducing tax deductions for housekeeping services in the United States and identify a Pareto-improving reform with marginal deduction rates that increase with service expenditures.

Our quantitative model has abstracted from a number of economic mechanisms that may affect the results. First, we have excluded any administrative costs and compliance costs of the reform. If those costs are taken into account, the introduction of deduction rules will be less beneficial. Second, deduction rules may induce households to eliminate any informal employment of household helpers in order to qualify for tax benefits. The actual welfare gains from deduction rules may thus be larger than the ones predicted by our model. However, the welfare consequences of informal work are generally ambiguous due to a discrepancy between efficiency and redistributive concerns (Doligalski and Rojas, 2019). Third, our analysis has abstracted from general equilibrium effects. A possible direction for future research would be to explore effects of deduction rules on the tax-exclusive prices of deductible goods and services. Another interesting, but very challenging extension would be to explore the role of preference heterogeneity. For instance, households may differ in their productivity for domestic work. In such a framework, the utility function would explicitly depend on non-market work (in addition to the dependency on service expenditures), which creates a moral hazard problem on top of the private information problem for skills. We leave an extension along those lines for future research.

A Appendix

A.1 Proofs

Proof of Proposition 1. We set up the augmented Hamiltonian

$$
\begin{aligned}
H =& f(n)\left[\delta - \sum_{k=1}^{K} \varepsilon - w^{-1}\left(U - v\left(d+\varepsilon, y+\delta; n\right); d+\varepsilon\right)\right] \\
&+ \lambda(n)\left[U - w(c,d) - v\left(d, y; n\right)\right] + \mu(n) v_n\left(d+\varepsilon, y+\delta; n\right)
\end{aligned}
$$

and derive the first-order conditions for ε and δ (evaluated at $\varepsilon = \delta = 0$):

$$
f\left[1 - v_d \frac{1}{w_c} - \frac{w_d}{w_c}\right] = \mu v_{nd} \tag{20}
$$

$$
f\left[1 + v_y \frac{1}{w_c}\right] = -\mu v_{ny} \tag{21}
$$

where we have used the derivatives of the inverse function, $w_c^{-1} = 1/w_c$ and $w_d^{-1} = -w_d/w_c$.

If $v_{nd} = 0$, we obtain

$$
\tau_d = 1 - \frac{w_d + v_d}{w_c} = 0. \tag{22}
$$

Otherwise, if $v_{nd} \neq 0$, we can divide the first-order conditions and obtain

$$
\frac{w_c - (v_d + w_d)}{w_c + v_y} = \frac{1 - \frac{v_d + w_d}{w_c}}{1 + \frac{v_y}{w_c}} = -\frac{v_{nd}}{v_{ny}}, \tag{23}
$$

which establishes Eq. (13).

■

Proof of Proposition 2. Under Assumption 1, the relationship between the cross derivatives of leisure utility imply

$$
-\frac{v_{nd}}{v_{ny}} = \frac{\frac{y}{n^2} E' \tilde{v}''}{\frac{y}{n^3}\tilde{v}'' - \frac{\tilde{v}'}{n^2}} = \frac{nE'}{1+e} \tag{24}
$$

where $e := -\frac{n\tilde{v}'}{y\tilde{v}''} > 0$. Moreover, using the definition of wedges we have

$$\frac{1-\tau_d}{1-\tau_y} = \frac{E'\tilde{v}'}{\frac{\tilde{v}'}{n}} = nE'. \tag{25}$$

Therefore, Eq. (14) implies

$$\frac{\tau_d}{1-\tau_d} = \frac{\tau_y}{1-\tau_y}\frac{1}{1+e}. \tag{26}$$

■

Proof of Proposition 3. We specialize the perturbation outlined in Section 3.1 for the time-endowment model. Suppose that time-saving investment d changes by an amount ε. Then, in order not to violate incentive compatibility, output y has to change by $\delta(\varepsilon)$ such that

$$\tilde{v}'\left(E(d) - \frac{y}{n}\right)\frac{y}{n^2} = \tilde{v}'\left(E(d+\varepsilon) - \frac{y+\delta(\varepsilon)}{n}\right)\frac{y+\delta(\varepsilon)}{n^2}. \tag{27}$$

Now, to ensure that the individual's utility is unaffected, the consumption has to be changed by $\gamma(\varepsilon)$ to ensure that

$$w(c) + \tilde{v}\left(E(d) - \frac{y}{n}\right) = w(c+\gamma(\varepsilon)) + \tilde{v}\left(E(d+\varepsilon) - \frac{y+\delta(\varepsilon)}{n}\right). \tag{28}$$

The resource gain of the perturbation is $R(\varepsilon) = -\varepsilon + \delta(\varepsilon) - \gamma(\varepsilon)$. We are particularly interested in the marginal resource gain: $R'(\varepsilon) = -1 + \delta'(\varepsilon) - \gamma'(\varepsilon)$.

Implicit differentiation of (28) yields $\gamma'(\varepsilon) = -(1-\tau_d^\varepsilon) + \delta'(\varepsilon)(1-\tau_y^\varepsilon)$, which implies that the marginal resource gain can be written as:

$$R'(\varepsilon) = -\tau_d^\varepsilon + \delta'(\varepsilon)\tau_y^\varepsilon. \tag{29}$$

Implicit differentiation of (27) yields:

$$\delta'(\varepsilon) = \frac{1-\tau_d^\varepsilon}{1-\tau_y^\varepsilon}\frac{1}{1+e^\varepsilon}, \tag{30}$$

where e^ε is the Frisch elasticity of labor supply with respect to the net-of-tax rate $1-\tau_y$ (holding time investment fixed).[19] Formally, we have

$$e^{\varepsilon} = -\frac{\tilde{v}'\left(E(d-\varepsilon) - l^{\varepsilon}\right)}{\tilde{v}''\left(E(d-\varepsilon) - l^{\varepsilon}\right) l^{\varepsilon}}, \quad \text{where } l^{\varepsilon} = \frac{y + \delta(\varepsilon)}{n}. \tag{31}$$

Overall, we can now write the resource gradient $R'(\varepsilon)$ as:

$$R'(\varepsilon) = -\tau_d^{\varepsilon} + \tau_y^{\varepsilon} \frac{1-\tau_d^{\varepsilon}}{1-\tau_y^{\varepsilon}} \frac{1}{1+e^{\varepsilon}}. \tag{32}$$

Starting from an allocation with $0 = \tau_d < \tau_y < 1$, Equation (32) for the marginal resource gain takes the following form:

$$R'(0) = -\tau_d + \tau_y \frac{1-\tau_d}{1-\tau_y} \frac{1}{1+e} = \tau_y \frac{1}{1-\tau_y} \frac{1}{1+e} > 0. \tag{33}$$

Hence, we obtain a resource gain by marginally increasing d, i.e., choosing $\epsilon > 0$. Using $\delta'(0) > 0$ and $\gamma'(0) < 0$, we note that the associated change of income is positive, whereas the associated change of consumption is negative. ■

Proof of Proposition 4. Starting from an allocation with $0 < \tau_d = \tau_y < 1$, Equation (32) for the marginal resource gain takes the following form:

$$R'(0) = -\tau_d + \tau_y \frac{1-\tau_d}{1-\tau_y} \frac{1}{1+e} = -\tau_y \frac{e}{1+e} < 0. \tag{34}$$

Hence, we obtain a resource gain by marginally reducing d, i.e., choosing $\epsilon < 0$. Using $\delta'(0) > 0$ and $\gamma'(0) < 0$, we note that the associated change of income is negative, whereas the associated change of consumption is positive. ■

A.2 Decentralization

Next, we justify our mechanism design approach to taxation and discuss how to implement an allocation as a competitive equilibrium with taxes. We demonstrate that any incentive-feasible allocation $(c(n), d(n), y(n))_{n \in \mathcal{N}}$ can be decentralized through a general (nonlinear and nonsep-

[19] In frameworks without time investment, this variable represents the standard Frisch elasticity of labor supply with respect to the net-of-tax rate. In the present environment, the concept becomes slightly more specific. We can interpret e^{ε} as the Frisch elasticity of labor supply with respect to the net-of-tax rate *holding time-saving investment fixed* or, equivalently, as the Frisch elasticity of labor supply with respect to the net-of-tax rate when time-saving investments are fully deductible from taxable income (not holding time-saving investment fixed). See Appendix A.3 for further details.

arable) income tax that depends on labor incomes and work-related expenses. Equivalently, there can be a labor income tax with a nonlinear, nonseparable deduction rule for work-related expenses.

A simple application of the taxation principle (Hammond, 1979; Rochet, 1985) implies that any incentive-feasible allocation can be implemented by a tax function $\bar{\mathcal{T}}(\cdot,\cdot)$ defined as

$$\bar{\mathcal{T}}(y(n),d(n)) := y(n) - d(n) - c(n),$$

and $\bar{\mathcal{T}}(y,d) := \infty$ for any pair (y,d) that is not part of the incentive-feasible allocation.[20]

In order to construct a less extreme implementation, note that a tax function $\mathcal{T}(\cdot,\cdot)$ implements the given allocation $(c(n),d(n),y(n))_{n\in\mathcal{N}}$ if and only if, for all n,

$$(c(n),d(n),y(n)) \in \arg\max_{c,d,y}\; u\,(c,d,y;n) \;\text{ s.t. }\; c = y - d - \mathcal{T}(y,d). \tag{35}$$

In fact, many functions $\mathcal{T}(\cdot,\cdot)$ exist that satisfy this set of conditions. What they need to satisfy for sure is $\mathcal{T}(y(n),d(n)) = \bar{\mathcal{T}}(y(n),d(n))$ for all n. We now derive the lower envelope of the set of tax schedules that satisfy (35) using an approach similar to that of Werning (2011), who studies the lower envelope of tax schedules in a framework with income and savings taxes. The lower envelope is least extreme in punishing choices that are not part of the incentive-feasible allocation.

Let us construct for each type n a function $\mathcal{T}_n(\cdot,\cdot)$ such that:

$$u\,(y - d - \mathcal{T}_n\,(y,d)\,,d,y;n) = u\,(c(n),d(n),y(n);n) \quad \forall\,(y,d)\,. \tag{36}$$

Note that this construction is possible if u is continuous and unbounded above and below in general consumption.[21] We know by construction that $\mathcal{T}_n(y(n),d(n)) = \bar{\mathcal{T}}(y(n),d(n))$, because otherwise Equation (36) would not hold for $(y,d) = (y(n),d(n))$. For this tax schedule, the agent of type n is indifferent between $(y\,(n)\,,d\,(n))$ and any other pair (y,d).

We claim that the upper envelope of the tax functions $\mathcal{T}_n$ implements the incentive-feasible

[20]By incentive compatibility, if the pair $(y(n),d(n))$ is part of the allocation, the associated level of general consumption $c(n)$ is unique.

[21]In particular, this approach does not rely on the separability assumption of Equation (1).

allocation. Define

$$\mathcal{T}^*(\cdot,\cdot) := \sup_n \mathcal{T}_n(\cdot,\cdot).$$

In this definition, the supremum is in fact a maximum because the type space is compact and $\mathcal{T}_n$ is continuous in n.

Proposition 5 (Implementation) *The tax function $\mathcal{T}^*$ implements the incentive-feasible allocation $(c(n), d(n), y(n))_{n\in\mathcal{N}}$. Moreover, if $\mathcal{T}$ is another tax function that implements the allocation, then $\mathcal{T} \geq \mathcal{T}^*$.*

Proof of Proposition 5. First, we claim

$$\mathcal{T}_{\hat{n}}(y(n), d(n)) \leq \mathcal{T}_n(y(n), d(n)) \text{ for all } \hat{n}, n.$$

Suppose, to the contrary, that there exist some $\hat{n}, n$ with

$$\mathcal{T}_{\hat{n}}(y(n), d(n)) > \mathcal{T}_n(y(n), d(n)).$$

Equivalently,

$$y(n) - d(n) - \mathcal{T}_{\hat{n}}(y(n), d(n)) < y(n) - d(n) - \mathcal{T}_n(y(n), d(n)) = c(n).$$

By the construction of $\mathcal{T}_{\hat{n}}$, we have

$$u\left(y(n) - d(n) - \mathcal{T}_{\hat{n}}\left(y(n), d(n)\right), d(n), y(n); \hat{n}\right) = u\left(c(\hat{n}), d(\hat{n}), y(\hat{n}); \hat{n}\right).$$

Hence, the previous inequality implies

$$u\left(c(n), d(n), y(n); \hat{n}\right) > u\left(c(\hat{n}), d(\hat{n}), y(\hat{n}); \hat{n}\right),$$

which violates the incentive compatibility constraint.

Hence, we have established $\mathcal{T}_{\hat{n}}(y(n), d(n)) \leq \mathcal{T}_n(y(n), d(n))$ for all $\hat{n}, n$. This implies

$$\mathcal{T}^*(y(n), d(n)) = \sup_{\hat{n}} \mathcal{T}_{\hat{n}}(y(n), d(n)) = \mathcal{T}_n(y(n), d(n)) \quad \forall n.$$

Moreover, by construction, the weak inequality $\mathcal{T}^*(y,d) \geq \mathcal{T}_n(y,d)$ holds for all pairs (y,d). Because agent n was indifferent between all pairs (y,d) under the tax system $\mathcal{T}_n$, it follows that the agent weakly prefers $(y(n),d(n))$ under the tax system $\mathcal{T}^*$.

Finally, let $\mathcal{T}$ be another tax function that implements the allocation. Suppose, to the contrary, that there exists some pair (y,d) with $\mathcal{T}(y,d) < \mathcal{T}^*(y,d)$. Then, by the definition of $\mathcal{T}^*$, there exists some n with

$$\mathcal{T}(y,d) < \mathcal{T}_n(y,d).$$

However, because $\mathcal{T}_n(y,d)$ was constructed to make the type n agent indifferent between (y,d) and $(y(n),d(n))$, the agent will strictly prefer (y,d) over $(y(n),d(n))$ under the tax system $\mathcal{T}$. This contradicts the assumption that $\mathcal{T}$ implements the allocation. ■

A.3 Frisch elasticity of labor supply in the time-endowment model

Consider an individual with skill n who maximizes utility subject to (locally) linear taxes and subsidies at rates t and s and a lump-sum transfer g. The decision problem is

$$\max_{c,d,l} \quad w(c) + \tilde{v}(E(d) - l) \quad \text{s.t.} \quad c + (1-s)\,d \leq (1-t)\,nl + g$$

Denoting the Lagrange multiplier for the budget constraint by λ, the first-order conditions of this problem are

$$\begin{aligned} w'(c) &= \lambda \\ E'(d)\tilde{v}'(E(d)-l) &= \lambda(1-s) \\ \tilde{v}'(E(d)-l) &= \lambda(1-t)\,n. \end{aligned}$$

Holding fixed the marginal utility of consumption λ and the time investment d, differentiation of the last equation with respect to $1-t$ yields

$$-\tilde{v}''(E(d)-l)\,\frac{\partial l}{\partial(1-t)} = \lambda n.$$

Therefore, the Frisch elasticity of labor supply (holding time investment fixed) is given by

$$e = \frac{\partial l}{\partial (1-t)} \frac{(1-t)}{l} = -\frac{\lambda n (1-t)}{\tilde{v}'' (E(d)-l)\, l} = -\frac{\tilde{v}' (E(d)-l)}{\tilde{v}'' (E(d)-l)\, l},$$

where we have used the first-order condition for labor supply.

Alternatively, consider a (locally) linear tax system where time investment is fully deductible from taxable income. Then the first-order conditions for time investment and labor supply are

$$\begin{aligned} E'(d)\tilde{v}' (E(d)-l) &= \lambda (1-t) \\ \tilde{v}' (E(d)-l) &= \lambda (1-t)\, n \end{aligned}$$

and differentiation (holding fixed λ) implies

$$\begin{aligned} E''\tilde{v}' \frac{\partial d}{\partial (1-t)} + E'\tilde{v}'' \left[E' \frac{\partial d}{\partial (1-t)} - \frac{\partial l}{\partial (1-t)} \right] &= \lambda \\ \tilde{v}'' \left[E' \frac{\partial d}{\partial (1-t)} - \frac{\partial l}{\partial (1-t)} \right] &= \lambda n. \end{aligned}$$

We substitute the second equation into the first and obtain

$$\frac{\partial d}{\partial (1-t)} = \frac{\lambda - E'\lambda n}{E''\tilde{v}'}.$$

Now the second equation yields

$$\frac{\partial l}{\partial (1-t)} = E'\lambda \frac{1-E'n}{E''\tilde{v}'} - \frac{\lambda n}{\tilde{v}''}.$$

Note that the first-order conditions for d and l imply $E' = \frac{1}{n}$. Hence, we obtain the Frisch elasticity of labor supply as

$$e = \frac{\partial l}{\partial (1-t)} \frac{(1-t)}{l} = -\frac{\lambda n (1-t)}{\tilde{v}'' l} = -\frac{\tilde{v}'}{\tilde{v}'' l}$$

where the last identity follows from the first-order condition for l.

A.4 Dynamic labor wedges

Kapicka (2020) studies Pareto-efficient labor taxes in a model of human capital formation through *learning by doing* or *learning or doing*. Similar to work-related consumption goods, intertemporal nonseparabilities of learning also generate a reason for differential taxation. By extending our theory to multiple work-related goods, we can relate to his findings.

Let the work-related good d_k represent the negative of output produced in period k and let y represesent the output in an initial period. Suppose that the preferences take the form $u = w(c) - V(z_0, z_1, \ldots, z_K)$, where V is increasing and convex, and labor supplies are given by $z_0 = y_0/n$ and $z_k = -d_k/n$ for $k \geq 1$. Then, $\tilde{\tau}_k := \tau_d^k$ represents the labor wedge at time k.

Note that the informational rents in this model are given by

$$v_n = \sum_{t=0}^{K} \frac{z_t}{n} V_t(z_0, z_1, \ldots, z_K).$$

Hence, the marginal effect of d_k on the informational rent is

$$v_{n,d_k} = -\frac{1}{n^2}\left(V_k + \sum_{t=0}^{K} z_t V_{t,k}\right),$$

which implies

$$\frac{v_{n,d_k}}{v_{n,d_{k'}}} = \frac{V_k + \sum_{t=0}^{K} z_t V_{t,k}}{V_{k'} + \sum_{t=0}^{K} z_t V_{t,k'}}.$$

The definition of the wedges implies

$$\frac{1-\tau_d^k}{1-\tau_d^{k'}} = \frac{V_k}{V_{k'}}.$$

Hence, we can rewrite the previous condition as

$$\frac{v_{n,d_k}}{v_{n,d_{k'}}} = \frac{1-\tau_d^k}{1-\tau_d^{k'}} \frac{1+\sum_{t=0}^{K} z_t \frac{V_{t,k}}{V_k}}{1+\sum_{t=0}^{K} z_t \frac{V_{t,k'}}{V_{k'}}}.$$

Therefore, Eq. (16) implies

$$\frac{\tau_d^k}{\tau_d^{k'}} = \frac{v_{n,d_k}}{v_{n,d_{k'}}} = \frac{1-\tau_d^k}{1-\tau_d^{k'}} \frac{1+\sum_{t=0}^{K} z_t \frac{V_{t,k}}{V_k}}{1+\sum_{t=0}^{K} z_t \frac{V_{t,k'}}{V_{k'}}}.$$

Equivalently,

$$\frac{\frac{\tilde{\tau}_k}{1-\tilde{\tau}_k}}{\frac{\tilde{\tau}_{k'}}{1-\tilde{\tau}_{k'}}} = \frac{1+\sum_{t=0}^{K} z_t \frac{V_{t,k}}{V_k}}{1+\sum_{t=0}^{K} z_t \frac{V_{t,k'}}{V_{k'}}}.$$

This condition describes the evolution of labor wedges across time and replicates Theorem 2 of Kapicka (2020). Kapicka also provides an insightful economic decomposition of this condition and shows how optimal labor wedges depend on effects of labor supply on current, past and future information rents. Moreover, Kapicka quantifies the different components of optimal labor wedges in a parametrized model for the U.S. economy.

A.5 Sufficient-statistics approach

Our theoretical results were derived with a mechanism-design approach in terms of the parameters of the underlying model. This contrasts with the sufficient-statistics approach (Chetty, 2009), where welfare statements are made solely in terms of empirically measurable concepts like elasticities and expenditure shares. The sufficient-statistics approach has the advantage that it relies less on structural assumptions. On the other hand, a general drawback is that even if one has empirical estimates of these sufficient statistics, these are typically local estimates that are mainly applicable to small policy reforms. Our goal is to think about larger reforms that move the economy from an inefficient allocation to the Pareto frontier, which calls for a more structural approach. By contrast, for a pure test of whether a given allocation is Pareto efficient, the sufficient-statistics approach seems most appropriate.

Next, we show that efficient taxes and subsidies can indeed be related in terms of sufficient statistics. It turns out that this condition will rely on four different elasticities. However, three of those elasticities are not standard concepts and at least two lack empirical evidence.

The sufficient-statistics approach can be most easily understood in a Ramsey taxation environment. Consider the optimization problem of a representative agent with utility $u(c, d, y)$ facing a linear labor income tax rate t_y and a linear subsidy on the work-related good t_d,

$$V(t_y, t_d) = \max_{y,d} \; u\left((1-t_y)y - (1-t_d)d, d, y\right).$$

The Ramsey planner solves

$$\max_{t_y, t_d} V(t_y, t_d) \ \text{ subject to } \quad - t_d d + t_y y \geq R,$$

where R is some exogenous revenue requirement.

The first-order condition for t_y (using the envelope theorem) is

$$-y + \lambda \left(y + t_y \frac{\mathrm{d}y}{\mathrm{d}t_y} - t_d \frac{\mathrm{d}d}{\mathrm{d}t_y} \right) = 0.$$

The first-order condition for t_d (using the envelope theorem again) is

$$d + \lambda \left(-d + t_y \frac{\mathrm{d}y}{\mathrm{d}t_d} - t_d \frac{\mathrm{d}d}{\mathrm{d}t_d} \right) = 0.$$

The first condition can be rewritten as

$$1 - \frac{u_c}{\lambda} = \frac{t_y}{1 - t_y} \varepsilon_{y,1-t_y} - \frac{t_d}{1 - t_y} \frac{d}{y} \varepsilon_{d,1-t_y}.$$

The second one we can write as

$$1 - \frac{u_c}{\lambda} = \frac{t_d}{1 - t_d} \varepsilon_{d,1-t_d} - \frac{t_y}{1 - t_d} \frac{y}{d} \varepsilon_{y,1-t_d}.$$

Putting both conditions together yields

$$\frac{t_d}{1 - t_d} \varepsilon_{d,1-t_d} - \frac{t_y}{1 - t_d} \frac{y}{d} \varepsilon_{y,1-t_d} - \frac{t_y}{1 - t_y} \varepsilon_{y,1-t_y} + \frac{t_d}{1 - t_y} \frac{d}{y} \varepsilon_{d,1-t_y} = 0,$$

where $\varepsilon_{a,b} = \frac{da}{db} \frac{b}{a}$ is the elasticity of variable a with respect to variable b. This condition only depends on sufficient statistics, i.e., four elasticities, income and expenditures for the work-related good. If we had knowledge about the values of all these elasticities, this condition would be very useful to test for the efficiency of a given allocation. Doerrenberg et al. (2017) is one of the few studies that provide direct evidence for $\varepsilon_{d,1-t_y}$. However, to the best of our knowledge, there is no evidence for the elasticities $\varepsilon_{d,1-t_d}$ and $\varepsilon_{y,1-t_d}$.[22]

[22]Hamilton (2018) makes some progress on this issue. However, his analysis rests on the assumption that the utilities of gross income and deductions are separable—an assumption that is violated for work-related goods.

To sum up, the sufficient-statistics approach does not seem well suited for our purpose because the relevant elasticities are not standard objects currently estimated in the empirical literature.

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