Optimal Redistribution: Rising Inequality vs. Rising Living Standards

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Abstract

Over the last decades, the United States has experienced a large increase in, both, income inequality and living standards. The workhorse models of optimal income taxation call for more redistribution as inequality rises. By contrast, living standards play no role for taxes and transfers in these homothetic environments. This paper incorporates living standards into the optimal income tax problem by means of non-homothetic preferences. In a Mirrlees setup, we show that rising living standards alter both sides of the equity-efficiency trade-off. As an economy becomes richer, non-homotheticities imply a fall in the dispersion of marginal utilities, which weakens distributional concerns but has ambiguous effects on efficiency concerns. In a dynamic incomplete-market setup calibrated to the United States in 1950 and 2010, we quantify this new channel. Rising living standards dampen by around 30% the desired increase in redistribution due to rising inequality.

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1 Introduction

Income inequality has been rising in the United States over the last decades, as documented in Piketty and Saez (2003), among others. As a result, fiscal redistribution has become a central topic in the policy debate, with popular calls for higher taxes and larger transfers. The literature on optimal income taxation characterizes the optimal tax-and-transfer ($t\&T$) system as a trade-off between equity and efficiency concerns. In the workhorse models, higher inequality indeed demands a more redistributive $t\&T$ system, as argued in Mankiw, Weinzierl, and Yagan (2009) and Diamond and Saez (2011).

Yet, in parallel to the rising income inequality, the United States has also experienced a very substantial increase in the standards of living. Mean income per capita has more than tripled since the 1950s, and the share of household expenditures spent on food has shrunk from more than 20% to less than 10%. Standard models of optimal taxation feature homothetic preferences and cannot generate the observed heterogeneity in consumption baskets, both in the cross-section and over time. Loosely speaking, they cannot capture how being poor in the 1950s differs from being poor in the 2010s. Therefore, these models shed no light on how rising living standards affect efficiency and distribution concerns, and thus the optimal $t\&T$ system.

This paper incorporates living standards into the optimal income tax problem by means of non-homothetic (NH) preferences—that is, preferences featuring heterogeneous income elasticities of demand across multiple goods. First, we analytically show how changes in living standards affect the equity-efficiency trade-off in a static Mirrlees (1971) setup with fully flexible nonlinear taxes. Second, we quantify the relative effects of rising living standards and rising inequality from 1950 to 2010, using two complementary approaches: the Mirrlees framework; and a rich dynamic incomplete-market setup with flexible yet parametric nonlinear taxes. We consistently find that rising living standards reduce the desired increase in redistribution due to rising inequality by around 30%, as measured by the difference in average income tax rates between the top- and bottom-income deciles.

Economic mechanisms. The curvature of the utility function is key to the optimal tax and transfer system, as it affects both distributional gains and efficiency concerns. Rising living standards alter the curvature of the utility function, an idea that is not new in the literature:

> *"Because they are close to subsistence, risk is [...] particularly painful to the poor." (Duflo 2006)*

> *"There exist at least two intuitive reasons why the IES might be smaller for the poor than it is for the rich. First, if there are positive subsistence consumption requirements, then*

poor consumers have a smaller portion of their budget left over after satisfying subsistence requirements to save or consume at their discretion. Second, the consumption in excess of subsistence of necessary goods (such as food) may be less substitutable across time than is the consumption of luxury goods. Since the poor spend a higher fraction of their total expenditure on subsistence and necessary goods than do the rich, their IES of total consumption expenditure may be smaller than the IES of the rich." (Atkeson and Ogaki 1996)

We show that this intuition holds with the two recent state-of-the-art NH preference specifications in the structural change literature, namely Comin, Lashkari, and Mestieri (2021) and Alder, Boppart, and Müller (2022). These preferences imply heterogeneous income elasticities across goods, such that the marginal spending composition of an additional dollar depends on the level of income. As an economy grows, the share of expenditures spent on necessities falls, capturing the rising living standards. These intratemporal spending allocation dynamics impose restrictions on the curvature of the utility function, with implications for intertemporal decisions as well: when further constrained by, e.g., labor supply dynamics, or by an empirically relevant level of risk aversion at one point in time, NH preferences imply decreasing relative risk aversion (DRRA)—a property consistent with ample empirical micro evidence.[1]

This property implies that rising living standards are not neutral for the equity-efficiency trade-off, with two main forces. First, dispersion in marginal utilities falls, which reduces the gains from redistributing resources from rich to poor households. As such, rising living standards weaken distributional concerns—a force we refer to as the *distributional gains* channel. Second, income effects weaken, which increases the efficiency costs of raising revenues but also decreases the efficiency costs of paying out transfers. As such, rising living standards have ambiguous effects on efficiency concerns—forces we refer to as the *efficiency costs* channel.

Two complementary approaches. These mechanisms are first formalized in a Mirrleesian setup. In particular, we consider fully nonlinear taxes in a static environment. We build on the analytical representation of optimal nonlinear taxes developed in Heathcote and Tsujiyama (2021) to formally decompose how living standards affect, both, efficiency costs and distributional gains of raising marginal tax rates along the income distribution. A calibration of this setup further allows us to quantify those different channels.

Second, we consider a dynamic incomplete-market setup. We follow a Ramsey approach and restrict the $t\&T$ system to belong to a flexible parametric class. While the Mirrleesian setup is powerful in imposing no restrictions on the $t\&T$ system, the dynamic environment

[1] See Section 4.3.4.

allows to discipline preferences from intra- and intertemporal choices, with a meaningful notion of risk aversion. In addition, it allows to separate income from expenditure distributions, crucial to disentangle efficiency from distributional concerns. The dynamic setup is further used to discipline the calibration of the Mirrleesian setup mentioned above.

The logic of the quantitative exercise is as follows. To focus on optimal dynamics of redistribution—rather than on its level—we follow an inverse optimum approach. That is, we calibrate the model to the U.S. economy in 1950, and derive Pareto weights which make the calibrated 1950 $t\&T$ system optimal. Then, we keep Pareto weights constant and compute the optimal $t\&T$ system for two cases. First, we only account for the rise in inequality until 2010, as a benchmark comparable to the literature. Second, we compute the optimal $t\&T$ system when also accounting for rising living standards. We interpret the difference in the optimal $t\&T$ systems as the standard-of-livings channel. We now describe the model calibration and preview the quantitative results.

Quantification. We calibrate the dynamic model to be consistent with key micro- and macro-level developments of the U.S. economy from 1950 to 2010, with the NH constant elasticity of substitution (CES) preferences of Comin et al. (2021) as our benchmark preference specification. Key for distributional concerns, the model is consistent with the dynamics of inequality. As for non-homotheticities, we use consumption and labor supply patterns in the cross-section and over time to discipline preference parameters, which eventually govern the degree of DRRA in the calibrated economy. Intertemporal decisions in the dynamic model allow further validation of the degree of DRRA given by the NH preferences. The implied degree of DRRA is modest, well within the range of plausible estimates from fields as diverse as development, consumption Euler equation estimation, and portfolio choice.

We then evaluate the effect of rising living standards on the optimal $t\&T$ system relative to the effect of rising inequality. In isolation, the large rise in inequality calls for a more redistributive $t\&T$ system. Accounting for the rise in living standards, the optimal $t\&T$ system still redistributes more in 2010 than in 1950, but to a lesser degree. Rising living standards reduce the optimal increase in redistribution due to rising inequality by about 30%, as measured by the difference in average income tax rates between the top- and bottom-income deciles.

We confirm that the calibrated Mirrlees setup delivers similar quantitative results, and further employ this setup for two purposes. First, we use the analytical income tax formula to quantify the different channels driving the effects of the rising living standards. We find that almost the whole effect stems from the *distributional gains* channel. Second, we conduct a series of robustness checks, using alternative calibrations with the NH CES preferences, as well as the preference specification of Alder et al. (2022). All experiments suggest effects of standards of living at least as large as in the benchmark.

Summing up, we consistently find that the rising living standards dampen by around 30% the desired increase in redistribution due to rising inequality, and most of this effect comes from weakening distributional concerns.

Related literature. Our work relates to both the public economics tradition, which studies optimal nonlinear income taxation (Heathcote and Tsujiyama 2021; Saez 2001), and the macroeconomic tradition, which focuses on restricted tax instruments in richer environments (Conesa and Krueger 2006; Heathcote, Storesletten, and Violante 2017). We connect these approaches with the notion of standards of living by incorporating changes in levels and NH preferences into the analysis.

In doing so, we contribute to an emerging literature on optimal taxes with NH preferences. Oni (2023) analyzes the optimal progressivity of a loglinear income tax function in a static general equilibrium model with non-homotheticies. In that setup, lower progressivity increases demand for luxuries; the relative price of necessities thus falls, which is beneficial for the poor. As a result, optimal progressivity falls, from 0.07 with homothetic preferences to 0.03 with NH preferences. Jaravel and Olivi (2024) consider NH preferences in a Mirrleesian income taxation problem and focus on the effects of heterogeneous inflation rates—that is, of changes in relative prices, with unequal incidence across the income distribution. In their setup, a rise in the price of necessities reduces optimal redistribution. Indeed, a rise in the price of a good reduces the value of a marginal dollar, as consumption baskets become more expensive. A rise in the price of necessities disproportionately affects the consumption baskets of the poor, reducing the value of redistributing a dollar from the rich to the poor. We adopt a different focus and analyze the effects of rising living standards, modeled as a homogeneous fall in all prices.[2]

More related in terms of motivation are the works of de Magalhaes, Martorell, and Santaeulàlia-Llopis (2022) and Tsujiyama (2022), which explore optimal taxation as an economy develops using NH preferences but abstracting from heterogeneous income elasticities across goods.[3]

Finally, our paper complements the literature addressing to what extent the rise in inequality in the United States justifies an increase in tax progressivity. Considering the United States between 1980 and 2016, Heathcote, Storesletten, and Violante (2020) find that the inequality channel is neutralized by increasing efficiency costs of tax progressivity resulting

[2]We also account for changes in relative prices to calibrate our quantitative model; Section 4.4 conducts a decomposition exercise to isolate the effects of relative price changes in our setup.

[3]de Magalhaes et al. (2022) show that, when considering private and public transfers, risk-sharing tends to decline with development, and discuss optimality of this finding in a one-good model with Stone-Geary preferences. Tsujiyama (2022) studies how subsistence self-employment, prevalent in developing economies, affects the equity-efficiency trade-off, also in a one-good environment.

from skill-biased technical change. Relatedly, Brinca, Duarte, Holter, and Oliveira (2022) reach a similar conclusion in a quantitative setup accounting for heterogeneous returns across occupations. Our paper puts into perspective the focus on changes in inequality, i.e. on second moments of the income distribution, by accounting for concurrent large changes in living standards, i.e. in first moments of the income distribution.

2 Modeling Rising Living Standards

We consider a continuum of heterogeneous households with labor productivity θ, which is distributed according to a probability density function $f(\theta)$ and cumulative distribution function $F(\theta)$. Households supply labor n and earn gross income $y = \theta n$. This results in expenditure $e = y - \mathcal{T}(y)$, where $\mathcal{T}$ captures the $t\&T$ system. Households allocate their expenditures to J different goods. We denote as $c = (c_1, \dots, c_J)$ the basket of consumption goods. We assume that utility is of the form $U(c) - Bn^{1+\varphi}/1+\varphi$, where $B > 0$ and φ^{-1} governs the Frisch elasticity. Additive separability allows to separate the labor choice from the consumption composition choice, and thus to decompose the optimization problem into two steps: (Step 1) solves for the optimal labor supply and yields the level of expenditure, while (Step 2) optimally allocates the expenditure across different goods:[4]

$$V(\theta; \mathcal{T}(\cdot), \Lambda, \overline{p}) \equiv \max_{e,n} u(e; \Lambda, \overline{p}) - B\frac{n^{1+\varphi}}{1+\varphi} \quad \text{s.t.} \quad e = n\theta - \mathcal{T}(n\theta), \tag{Step 1}$$

$$u(e; \Lambda, \overline{p}) \equiv \max_{\{c_j\}_j} U(c) \quad \text{s.t.} \quad \sum_j p_j c_j = e, \quad \text{where} \quad p_j \equiv \frac{\overline{p}_j}{\Lambda}. \tag{Step 2}$$

The vector of prices p is defined as relative prices $\overline{p}$ divided by a level parameter Λ. Relative prices are assumed constant throughout this section. Instead, we consider economies with different values for Λ, which homogeneously scales prices. Thus, we refer to Λ as the *level* of the economy. With NH preferences, a higher Λ implies a shift of consumption baskets away from necessities. This is what we define as the rising living standards.

Importantly, Λ and $\overline{p}$ only affect the labor supply decision through their impact on $u_e(e; \Lambda, \overline{p})$. This insight is useful to analyze the implications of Λ on the optimal $t\&T$ system in Section 3. Once we characterize the properties of $u(e; \Lambda, \overline{p})$, we can focus on (Step 1).

Section 2.1 introduces the two NH preferences we consider throughout the paper. Section 2.2 characterizes implications of $U(c)$ on the curvature of $u(e; \Lambda, \overline{p})$.

[4]The additive separability also implies that the Atkinson-Stiglitz theorem holds in this environment. Hence, the optimal tax system implies uniform commodity taxes.

2.1 Heterogenous Expenditure Elasticities

We measure rising living standards by changes in consumption baskets as an economy grows. There is ample evidence that Engel curves, depicting how spending on different goods varies with income, are not linear, and consumption baskets are heterogeneous—both over time and in the cross-section.[5] In other words, expenditure elasticities of demand are heterogeneous across goods. To capture the rising living standards, we thus consider utilities satisfying the following assumption.

Assumption 1. *Assume that $U(c)$ is such that expenditure elasticities are heterogeneous across goods. That is,*

$$\frac{\partial \log c_i}{\partial \log e} \neq \frac{\partial \log c_j}{\partial \log e} \text{ when } i \neq j.$$

There exist different functional forms for $U(c)$ that are consistent with this assumption. We focus on the two recent state-of-the-art NH preference specifications in the structural change literature, namely Comin et al. (2021) and Alder et al. (2022).

Case 1. *(NH CES Preferences)*

We first describe the NH CES preferences that go back to Hanoch (1975) and have been introduced into a multi-sector growth model by Comin et al. (2021). NH CES preferences are defined over the basket of consumption goods c by $U(c) = \mathbb{C}(c)^{1-\gamma}/(1-\gamma)$, where the consumption aggregator $\mathbb{C}(c)$ is implicitly defined as:

$$\sum_{j}^{J} \left(\Omega_j \mathbb{C}(c)^{\varepsilon_j}\right)^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} = 1, \tag{1}$$

with $\gamma \geq 0$, and $\sigma > 0$, $\Omega_j > 0\ \forall j$, $\epsilon_j > 0$ ($\epsilon_j < 0$) if $\sigma < 1$ ($\sigma > 1$) $\forall j$. Preferences collapse to a homothetic CES when $\varepsilon_j = 1 - \sigma\ \forall j$.

For this utility function, one obtains the following elasticities of consumption w.r.t. expenditure:

$$\frac{\partial \log c_j}{\partial \log e} = \sigma + (1-\sigma)\frac{\varepsilon_j}{\bar{\varepsilon}},$$

where $\bar{\varepsilon} = \sum_{j=1}^{J} \omega_j \varepsilon_j$ and ω_j are the expenditure shares of the different goods.[6] Goods j with $\varepsilon_j < \bar{\varepsilon}$ are necessities, as their expenditure elasticities are lower than unity. Goods j with $\varepsilon_j > \bar{\varepsilon}$ are luxuries. As opposed to Stone-Geary preferences, non-homotheticities do not vanish: differences in $\partial \log c_j / \partial \log e$ also prevail as e keeps growing.

[5] See Boppart (2014) and Herrendorf, Rogerson, and Valentinyi (2014), among many others.

[6] Comin et al. (2021) show that $\{\varepsilon_j\}$ can be rescaled by any positive scalar k: an allocation consistent with elasticities $\{\varepsilon\}$ will also be consistent with elasticities $\{k\varepsilon\}$. See Appendix A.1.1 for more details.

Comin et al. (2021) show that one can express the expenditure function as

$$e = \left(\sum_j \Omega_j \mathcal{C}(e; \Lambda, \overline{p})^{\varepsilon_j} \left(\overline{p}_j / \Lambda \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{2}$$

which defines the optimal consumption aggregator $\mathcal{C}(e; \Lambda, \overline{p})$ in terms of e.

Assumption 2. *Let $\sigma < 1$, so that goods are gross complements.*

For the rest of the paper, we focus on the case $\sigma < 1$, appropriate to capture changes in consumption baskets across broad sectors reflecting rising living standards.

Case 2. *(Intertemporally Aggregable (IA) Preferences)*

The second state-of-the-art NH preferences are the IA preferences introduced by Alder et al. (2022). These preferences are directly defined over expenditure:

$$u(e; \Lambda, \overline{p}) = \frac{1-\iota}{\iota} \left(\frac{1}{\mathbf{B}(p)} \left(e - \underbrace{\sum_j p_j \bar{c}_j}_{\mathbf{A}(p)} \right) \right)^{\iota} - \mathbf{D}\left(p\right), \tag{3}$$

where $\mathbf{B}(p) = \left(\sum_j \Omega_j p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ with $\sigma > 0, \sum_{j \in J} \Omega_j = 1$, and $\Omega_j \geq 0 \; \forall j$; $\mathbf{D}(p)$ homogenous of degree zero; and $\iota \in (0, 1)$. IA preferences are homothetic when $\mathbf{A}(p) = 0$ and $\mathbf{D}(p) = 0$.

Alder et al. (2022) show that these preferences allow for intertemporal aggregation, and nest both: generalized Stone-Geary (Herrendorf et al. 2014), through the $\mathbf{A}$ term; and price independent generalized linearity (PIGL) preferences (Boppart 2014), through the $\mathbf{D}$ term.

2.2 Non-Homotheticities and Marginal Utilities

We now derive implications of heterogeneous expenditure elasticities on the curvature of the indirect utility function $u(e; \Lambda, \overline{p})$, which contains all properties of non-homotheticities relevant for the optimal income taxation problem. We consider time-separable preferences for which the intertemporal elasticity of substitution (IES) is the inverse of RRA. Key to our analysis is to characterize how the curvature of static utility changes as expenditure grows.

Homothetic Benchmark. We first consider homothetic parameterizations of NH CES and IA preferences. NH CES preferences are homothetic when $\varepsilon_i = 1 - \sigma \; \forall i$—see equation (1)—and then feature CRRA with relative risk aversion γ. IA preferences are homothetic when $\mathbf{A}(p) = \mathbf{D}(p) = 0$—see equation (3)—and then feature CRRA with relative risk aversion $1 - \iota$. Starting from this natural homothetic isoelastic CRRA benchmark, we now look at the effect of non-homotheticities on relative risk aversion.

NH CES preferences and DRRA. Let $\gamma(e;\Lambda,\overline{p})$ denote the coefficient of relative risk aversion at expenditure e. With NH CES preferences, risk aversion depends on both the consumption aggregator $\mathcal{C}$ function and the curvature parameter γ:[7]

$$\gamma(e;\Lambda,\overline{p}) \equiv -\frac{u_{ee}e}{u_e} = \gamma\frac{\mathcal{C}_e(e;\Lambda,\overline{p})e}{\mathcal{C}(e;\Lambda,\overline{p})} - \frac{\mathcal{C}_{ee}(e;\Lambda,\overline{p})e}{\mathcal{C}_e(e;\Lambda,\overline{p})}, \tag{4}$$

where $\mathcal{C}_e$ and $\mathcal{C}_{ee}$ can be obtained through implicit differentiation of the expenditure function. The first term in (4) multiplies the curvature parameter γ with the elasticity of $\mathcal{C}$ with respect to expenditures. The second term captures the elasticity of $\mathcal{C}_e$ with respect to expenditures.

Under a homothetic parameterization ($\varepsilon_i = \varepsilon_j \ \forall(i,j)$), $\mathcal{C}(e;\Lambda,\overline{p}) \propto e$ and the elasticity of $\mathcal{C}$ w.r.t. e is 1 while the elasticity of $\mathcal{C}_e$ w.r.t. e is 0. Thus, equation (4) simplifies to $\gamma(e;\Lambda,\overline{p}) = \gamma$ and preferences feature CRRA.

Under a non-homothetic parameterization ($\varepsilon_i \neq \varepsilon_j$), we show in Lemma 1 that the elasticity of $\mathcal{C}$ w.r.t. expenditures is unambiguously decreasing in expenditures.

Lemma 1. *The elasticity of $\mathcal{C}$ w.r.t. e is strictly decreasing iff $\exists i : \varepsilon_i \neq \varepsilon_j$. That is,*

$$\varepsilon_i \neq \varepsilon_j \Rightarrow \frac{\partial}{\partial e}\left(\frac{\mathcal{C}_e(e;\Lambda,\overline{p})e}{\mathcal{C}(e;\Lambda,\overline{p})}\right) < 0 \quad \forall e.$$

Proof. See Appendix A.1.1. □

This force captures the rising living standards: at low levels of expenditures, an increase in expenditure is more strongly directed towards necessities, translating into a larger increase in the consumption aggregator $\mathcal{C}$. Thus, the first term in (4) generates a force towards DRRA, which is further amplified by the curvature parameter γ: the larger γ, the steeper is the fall of relative risk aversion with expenditure.

The second term in (4), the elasticity of $\mathcal{C}_e$ w.r.t. e may be increasing or decreasing in e, depending on the parameterization of the utility function and reflecting the flexibility of the NH CES preferences. Yet, that term is independent of γ. As such, given $\mathcal{C}$, there always exists a γ large enough for preferences to feature DRRA.

The parameters $\{\varepsilon_j\}$ and σ can be estimated from intratemporal allocations of expenditure across goods, thereby disciplining $\mathcal{C}$. Given $\mathcal{C}$, the curvature parameter γ pins down the entire schedule of relative risk aversion in (4) and thus the DRRA property of preferences. How can γ be disciplined? The level of risk aversion at one point in time determines γ. Alternatively, γ maps into other observables, such as income effects or labor supply decisions.

[7]As discussed in footnote 6, $\{\varepsilon\}$ can be rescaled by any positive scalar. Appendix A.1.1 shows that relative risk aversion, as measured in (4), as well as labor supply decisions, are invariant to the rescaling.

One can further formalize this link between DRRA and labor supply dynamics in the analytical NH CES setup of Bohr, Mestieri, and Yavuz (2023) described in Assumption 3.

Assumption 3. *Consider a continuum of goods, with elasticities* $\{\varepsilon_j\}_{j\in[0,1]}$ *following a* Γ *distribution. Let prices* $\{p_j\}$ *and taste parameters* $\{\Omega_j\}$ *be log-affine functions of elasticities.*

Proposition 1. *Under Assumptions 2-3, NH CES preferences satisfy DRRA iff* $\gamma - 1 > 0$.

Corollary 1. *Under Assumptions 2-3, NH CES preferences satisfy DRRA iff labor supply falls with rising living standards.*

Proof. See Appendix A.1.1. □

In this analytically tractable setup, there is a one-to-one relationship between DRRA and falling labor supply, a robust empirical pattern we discuss in Section 4.3.2. In Section 4.2, when calibrating a model with three sectors, we target a standard level of risk aversion in 2010, which quantitatively generates both a fall in labor supply and a fall in relative risk aversion.

IA preferences and DRRA.

Proposition 2. *IA preferences satisfy DRRA iff* $\mathbf{A}(p) > 0$.

Corollary 2. *If IA preferences generate falling labor supply with rising living standards, they satisfy DRRA.*

Proof. See Appendix A.1.2. □

For IA preferences, the DRRA property emerges from the subsistence term $\mathbf{A}$. When $\mathbf{A}(p) = 0$ (PIGL), preferences feature CRRA. $\mathbf{A}(p) > 0$ implies DRRA, and it is also a necessary condition for labor supply to fall over time, as measured in the data.[8] In the quantification of the IA preferences in Section 5.3, the curvature parameter is calibrated to match the same long-run relative risk aversion as under NH CES preferences, and the calibration of the $\{\bar{c}_j\}$ is such that $\mathbf{A}(p) > 0$: preferences feature, both, DRRA and falling labor supply over time.

2.3 Cardinalization

Section 2.2 argues that intratemporal allocations—consumption and labor supply policies—are informative about intertemporal properties of the utility function. Doing so, we echo a longstanding theoretical literature that links intra to intertemporal allocations. Stiglitz

[8] Alder et al. (2022) also reject the PIGL specification in favor of IA preferences with $\mathbf{A}$ and $\mathbf{D}$ not equal to 0, with an estimation purely based on consumption baskets.

(1969) proves that the observed heterogeneity in consumption baskets along the income dimension rules out risk neutrality.[9] Crossley and Low (2011) prove that the observed rank of demand systems rules out the constant relative risk aversion implied by both power preferences over total expenditures and PIGL preferences.[10]

Going beyond rejecting CRRA based on intratemporal allocations requires to take a stand on cardinalization. Indeed, in principle, any monotonic transformation $V(.)$ of intratemporal utility $U(c) - n^{1+\varphi}/(1+\varphi)$ would leave intratemporal allocations unchanged while altering intertemporal properties of the utility function. Browning and Crossley (2000) derive a further characterization by assuming additively separable goods. They show that the higher expenditure elasticity of luxuries also implies an increasing IES. They prove an equivalence between luxuries being easier to reduce when cutting expenditure down, and "luxuries [being] easier to postpone", as they title their paper, under this additivity restriction.

In this Section, we extended this logic to the two state-of-the-art NH preferences, for which goods are not additively separable, using intratemporal allocations of goods and labor supply. Doing so, we implicitly assumed an identity cardinalization—i.e. $V(.) = \text{id}(.)$ the identity function.[11] This cardinalization is supported by the quantitative model developed in Section 4. Intertemporal household decisions in this dynamic model compare well to their data counterpart, thereby validating the choice of $V(.) = \text{id}(.)$. Marginal propensities to consume (MPC), wealth effects, and wealth distributions are well aligned with available data in 2010. We further maintain the cardinalization over time—that is, we assume that $V_{1950}(.) = V_{2010}(.)$—as implicitly done in the literature evaluating optimal fiscal policy over time (Brinca et al. 2022; Heathcote et al. 2020; Lockwood and Weinzierl 2016).

This cardinalization implies DRRA, a property for which there is strong support both empirically and conceptually.

Empirically, there is direct evidence of varying RRA or IES. Ogaki and Zhang (2001) and Zhang and Ogaki (2004) estimate DRRA using consumption data from Pakistani and Indian villages. Atkeson and Ogaki (1996) estimate increasing IES both using Indian panel data and in the aggregate time series for India and the United States, as do Blundell, Browning, and Meghir (1994) and Attanasio and Browning (1995) with UK data.[12] In addition to these direct estimates, DRRA is an important feature in making theory consistent with data in

[9]See also Hanoch (1977).

[10]Quoting their conclusion: *"The importance of our result is in refuting the belief that properties of intertemporal allocations can be independent of the properties of within-period allocation. This belief underpins the use of the constant-IES assumption in much modern macroeconomics."* (Crossley and Low 2011, p.104).

[11]Appendix A.1.3 shows how the main theoretical result can extend beyond identity, as long as $V(.)$ does not feature too strongly increasing *absolute* risk aversion (ARA)—recall that increasing ARA implies that wealthier households hold less risky assets, in absolute terms.

[12]See Section 4.3.4 for more details on the empirical findings.

a variety of fields.[13] Yet, RRA is difficult to measure precisely. Intratemporal allocations provide guidance on the magnitude of the fall in RRA as the economy grows.

Conceptually, the link between non-homotheticities and DRRA makes a lot of economic sense. As argued in Atkeson and Ogaki (1996) or in Duflo (2006), the poor may suffer more from risk as they are closer to subsistence.

Taking stock. NH preferences generally feature DRRA, a property which will be key to analyze how optimal $t\&T$ systems vary with living standards. As living standards rise, the dispersion of marginal utilities will fall under DRRA, altering both distributional gains and efficiency costs of taxation.

3 Optimal Policy: Theoretical Analysis

We consider a social planner that assigns Pareto weights $w(\theta)$ to households of type θ and optimally chooses a fully nonlinear $t\&T$ system $\mathcal{T}(\cdot;\Lambda)$ in the spirit of Mirrlees (1971). We analyze how rising living standards alter the optimal tax schedule—holding fixed relative prices, the distribution of skills, and the Pareto weights. Section 3.1 formally shows that optimal taxes are independent of the level of the economy Λ when preferences are homothetic. Section 3.2 formalizes the different effects of changes in Λ on optimal taxes when preferences are NH. Section 3.3 considers a special case which allows to sign the overall effect: starting from the Laissez-Faire, rising living standards unambiguously call for less redistribution.[14]

The government's problem is given by

$$\max_{\mathcal{T}(\cdot;\Lambda)} \int_{\underline{\theta}}^{\overline{\theta}} V(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda)w(\theta)f(\theta)d\theta \quad \text{s.t.} \quad \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{T}(n(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda)\theta;\Lambda)f(\theta)d\theta \geq G,$$

subject to optimal household behavior given the tax function:

$$n(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda) \equiv \arg\max_{e,n} u(e;\Lambda) - B\frac{n^{1+\varphi}}{1+\varphi} \quad \text{s.t.} \quad e = n\theta - \mathcal{T}(n\theta;\Lambda),$$

where G denotes exogenous spending, $V(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda)$ is defined in (Step 1), and we suppress

[13] In growth theory, Rebelo (1992), Ogaki, Ostry, and Reinhart (1996), and Chatterjee and Ravikumar (1999) argue that non-homotheticities induced by minimum consumption requirements can explain low savings rates in poor countries. In consumption theory, NH preferences that imply DRRA can account for consumption responses to permanent income changes (Straub 2019). In finance, DRRA helps in matching portfolios across the wealth distribution (Cioffi 2021; Wachter and Yogo 2010) and in mitigating the equity premium puzzle (Ait-Sahalia, Parker, and Yogo 2004). In development, Donovan (2021) argues that DRRA is important in accounting for aggregate productivity differences across countries.

[14] In a similar formal environment, Jaravel and Olivi (2024) consider a different question: the effects of heterogenous inflation rates on optimal income taxes. For that purpose, they locally assume $u_{ee} = 0$ at initial prices for most of their analysis, so that a uniform change in prices has no effect on optimal redistribution by construction.

the constant relative price vector p as an argument of tax and policy functions for readability. To ease notation we also replace $u_e(e(\theta;\Lambda);\Lambda)$ with $u_e(\theta;\Lambda)$, and omit $\mathcal{T}(\cdot;\Lambda)$ as argument of household policy functions when possible throughout this section.

We now state the solution to the optimal tax problem in the following lemma.

Lemma 2. *For each type θ^*, the optimal marginal tax rate $\mathcal{T}'(y(\theta^*;\Lambda);\Lambda)$ is characterized by $E(\theta^*;\mathcal{T},\Lambda) = D(\theta^*;\mathcal{T},\Lambda)$, where:*

$$E(\theta^*;\mathcal{T},\Lambda) = 1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*;\Lambda);\Lambda)}{1-\mathcal{T}'(y(\theta^*;\Lambda);\Lambda)}\frac{1}{1+\varphi}\frac{\theta^* f(\theta^*)}{1-F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta;\Lambda);\Lambda)\eta(\theta;\Lambda)\frac{dF(\theta)}{1-F(\theta^*)}}{1 + \int_{\underline{\theta}}^{\bar{\theta}} \mathcal{T}'(y(\theta;\Lambda);\Lambda)\eta(\theta;\Lambda)dF(\theta)},$$

$$D(\theta^*;\mathcal{T},\Lambda) = 1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta;\Lambda)w(\theta)\frac{dF(\theta)}{1-F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta;\Lambda)w(\theta)dF(\theta)},$$

and income effects of type-θ worker $\eta(\theta;\Lambda) \equiv dy(\theta;\Lambda)/d\mathcal{T}(0;\Lambda)$ are given by

$$\eta(\theta;\Lambda) = \frac{\gamma(\theta;\Lambda)\frac{y(\theta;\Lambda)}{e(\theta;\Lambda)}}{\varphi + \gamma(\theta;\Lambda)\frac{y(\theta;\Lambda)}{e(\theta;\Lambda)}(1-\mathcal{T}'(y(\theta;\Lambda);\Lambda)) + \frac{\mathcal{T}''(y(\theta;\Lambda);\Lambda)y(\theta;\Lambda)}{1-\mathcal{T}'(y(\theta;\Lambda);\Lambda)}}, \tag{5}$$

where $\gamma(\theta;\Lambda) \equiv \gamma(e(\theta;\Lambda);\Lambda)$ to ease notation.

Proof. See Appendix A.2.2. □

This derivation is a standard exercise.[15] As in Heathcote and Tsujiyama (2021), we characterize the optimal marginal tax rate at income $y(\theta^*,\Lambda)$ as the one equalizing distributional gains $D(\theta^*;\mathcal{T},\Lambda)$ to efficiency costs $E(\theta^*;\mathcal{T},\Lambda)$.

Distributional gains. $D(\theta^*;\mathcal{T},\Lambda)$ captures the distributional gains from increasing the marginal tax at income $y(\theta^*;\Lambda)$ and redistributing the additional revenues lump-sum. The numerator in the fraction captures the utility loss from the higher taxes paid by workers of type $\theta \geq \theta^*$. The denominator in the fraction captures the utility gain from the larger lump-sum transfer to all workers.

When all workers are identical and Pareto weights are equalized, $D(\theta^*;\mathcal{T},\Lambda) = 0$: there is no gain from redistributing. Instead, with heterogeneous workers, the average marginal utility of workers above θ^*, in the numerator, is typically lower than the average marginal utility across the entire distribution, in the denominator. Thus, $D(\theta^*;\mathcal{T},\Lambda) > 0$: there are positive gains from redistributing. The larger the dispersion in marginal utilities u_e, the larger the term $D(\theta^*;\mathcal{T},\Lambda)$ becomes.

[15] This result can be derived with a mechanism-design approach, or as formalized in Golosov, Tsyvinski, and Werquin (2014), with a tax-reform approach. We follow the latter.

Efficiency costs. $E(\theta^*; \mathcal{T}, \Lambda)$ captures the efficiency costs from increasing the marginal tax at income $y(\theta^*; \Lambda)$ and redistributing the additional revenues lump-sum. The numerator in the fraction captures the efficiency cost of raising revenue from households with $\theta \geq \theta^*$, which depends on two forces: (i) with positive Frisch elasticity $1/\varphi > 0$, workers with type θ^* reduce labor supply in response to the higher marginal tax; (ii) with positive income effects $\eta(\cdot; \Lambda) > 0$, workers with type $\theta > \theta^*$ increase their labor supply in response to the higher average tax rate. The denominator in the fraction captures the efficiency cost of redistributing the additional revenues: with positive income effects, all workers decrease their labor supply in response to the larger lump sum. Hence, larger income effects have ambiguous effects: they lower the efficiency cost of raising taxes in the numerator, but increase the efficiency costs of raising the lump-sum transfer in the denominator.

3.1 Benchmark: Homothetic Preferences

We first consider as a benchmark the homothetic parameterizations of Preferences 1 and 2. As discussed in Section 2.2, they satisfy CRRA. Proposition 3 formally states the irrelevance of living standards for the optimal $t\&T$ system.

Proposition 3. *Assume preferences $u(e; \Lambda)$ satisfy CRRA in Preferences 1 and 2. Then, $D_\Lambda(\theta, \Lambda) = E_\Lambda(\theta, \Lambda) = 0$; and expenditures and incomes grow with Λ at constant rate $\alpha \equiv (1-\gamma)/(\varphi+\gamma)$ $\forall \theta$.*

Corollary 3. *When preferences satisfy CRRA, optimal marginal and average tax rates are independent of Λ $\forall \theta$; and relative incomes are independent of Λ.*

Proof. See Appendix A.2.3. □

To provide intuition on the irrelevance of living standards for the optimal $t\&T$ system, we build on the optimal tax formula in Lemma 2.[16] First, when marginal rates are constant, it is easy to show that incomes grow at a rate α which can be positive or negative depending on the relative strength of income and substitution effects, but which is constant across θ. As such, income inequality is constant. With constant marginal rates, tax payments grow at the same rate as income, and so do the lump-sum transfer and expenditures. As such, expenditure inequality is constant.

We then argue that, with constant inequality, both distributional gains $D(\theta, \Lambda)$ and efficiency costs $E(\theta, \Lambda)$ are unchanged with living standards, such that constant marginal rates are indeed optimal. We start with distributional gains. As expenditure inequality is constant, ratios of marginal utilities across households are unchanged with Λ under CRRA

[16] Appendix A.2.3 characterizes the income-tax reform that implements the optimal $t\&T$ described in Proposition 3.

preferences. Thus, the welfare gains from redistributing from workers above θ^* to those below θ^* are unaffected. We turn to efficiency costs. Income effects depend on Frisch elasticity, risk aversion, income-to-expenditure ratios, as well as marginal tax rates themselves. As all are constant, income effects do not vary with Λ and efficiency costs are unaffected.

Summing up, with homothetic CRRA parameterizations, which imply constant expenditure shares thus abstracting from the rise in living standards, changes in levels leave both marginal and average $t\&T$ rates unchanged.

3.2 Accounting for living standards

We now consider the NH parameterizations of Preferences 1 and 2 which satisfy the DRRA property. A change in the level of the economy Λ alters the optimal $t\&T$ system through three channels: (*i*) a *distributional gains* channel; (*ii*) an *efficiency costs* channel; and (*iii*) an *income distribution* channel.

We assume that marginal tax rates remain constant for each type θ as living standards rise, as would be optimal under homothetic preferences. We show how rising living standards change (*i*) marginal utilities; (*ii*) income effects; and (*iii*) relative income and expenditure inequality, altering the efficiency-redistribution trade-off and thus optimal tax rates. We examine each channel separately next. All proofs are relegated to Appendix A.2.4.

Distributional gains channel. The distributional gains channel calls for less redistribution as living standards rise. With non-homothetic preferences, the poor shift their consumption away from necessities, resulting in marginal utilities falling faster for the poor than for the rich. Thus, at fixed relative inequality in expenditure, the dispersion of marginal utilities gets compressed, lowering the distributional gains of taxes.

Proposition 4. *Assume preferences $u(e;\Lambda)$ satisfy DRRA in Preferences 1 and 2. Fixing allocations and taxes, rising living standards decrease distributional gains $D_\Lambda(\theta,\Lambda) < 0$.*

Efficiency costs channel. The efficiency cost channel depends on living standards through income effects. Intuitively, households close to their subsistence levels strongly increase their labor supply when tax payments increase. This force weakens as living standards rise, with ambiguous effects on the efficiency costs channel.

With general nonlinear tax functions, one needs to impose one further assumption to formally establish that income effects weaken with rising living standards.

Assumption 4. *Let the optimal $t\&T$ system $\mathcal{T}(.;\Lambda)$ be such that $\frac{\mathcal{T}''(y(\theta;\Lambda);\Lambda)y(\theta;\Lambda)}{1-\mathcal{T}'(y(\theta;\Lambda);\Lambda)} \geq -\varphi$.*

Assumption 4 essentially imposes a restriction on Pareto weights, so that the optimal $t\&T$ system at Λ does not imply marginal rates that are too strongly decreasing in types.

For instance, assuming a loglinear function $\mathcal{T}(y) \equiv y - \lambda y^{1-\tau}$, Assumption 4 boils down to $\tau \geq -\varphi$.

Lemma 3. *Assume preferences $u(e;\Lambda)$ satisfy DRRA in Preferences 1 and 2. Under Assumption 4, fixing allocations and taxes, rising living standards weaken income effects.*

Lower income effects increase the efficiency costs of raising revenues, calling for less redistribution as living standards rise, as workers with type $\theta > \theta^*$ increase their labor supply by less in response to the higher average tax rate. In contrast, lower income effects reduce the efficiency costs of redistributing revenues, calling for more redistribution as living standards rise, as all workers decrease their labor supply by less in response to the larger lump sum. As such, the overall effect of rising living standards on efficiency costs of redistribution is ambiguous.

Proposition 5. *Assume preferences $u(e;\Lambda)$ satisfy DRRA in Preferences 1 and 2. Fixing allocations and taxes, rising living standards: (i) increase efficiency costs due to weaker income effects to average tax rates; (ii) decrease efficiency costs due to weaker income effects to lump sum transfers.*

Having characterized how distributional gains and efficiency gains change with rising living standards under constant taxes and relative incomes, we now discuss how rising living standards alter relative incomes.

Income distribution channel. Rising living standards induce households to adjust their labor supply. With DRRA, as low-θ workers move away from their subsistence level, they typically reduce their hours by more—or increase them by less—than high-θ workers. Thus, relative income inequality increases with living standards, which calls for more redistribution. Formally establishing this result is difficult without a functional form for $\mathcal{T}$, as labor supply decisions also depend on the shape of the marginal tax rate schedule. We consider two standard cases: the loglinear tax function (Heathcote et al. 2017), and the Laissez-Faire—which, with all marginal rates at zero, is a special case of the loglinear tax function.

Proposition 6. *Assume preferences $u(e;\Lambda)$ satisfy DRRA in Preferences 1 and 2. Assume the t&T system $\mathcal{T}(.;\Lambda)$ satisfies: (i) the Laissez-Faire $\mathcal{T}(y;\Lambda) = 0\ \forall y$; or (ii) the loglinear tax function $\mathcal{T}(y;\Lambda) = y - \lambda y^{1-\tau}$. Then, keeping taxes constant, low-type workers increase (decrease) their labor supply by less (more) than high-type workers:*

$$\frac{\partial}{\partial \theta}\left(\frac{n_\Lambda(\theta,\Lambda;\mathcal{T}(.,\Lambda))}{n(\theta,\Lambda;\mathcal{T}(.,\Lambda))}\right) > 0.$$

To sum up, rising living standards affect the efficiency-redistribution trade-off in three ways: (i) *distributional gains* decrease with rising living standards, calling for less redistribution; (*ii*) effects on *efficiency costs* are ambiguous; and (*iii*) the *income distribution* becomes more unequal, calling for more redistribution.

3.3 Laissez-Faire

We can characterize analytically the sign of the total effect of rising living standards starting from the special case of the Laissez-Faire, in which the distribution of Pareto weights is such that marginal and average tax rates are zero at a particular level Λ. Proposition 7 establishes that the *distributional gains* channel dominates, so that rising living standards unambiguously decrease optimal redistribution.

Proposition 7. *Consider an economy at a level Λ with a distribution of Pareto weights implementing the Laissez-Faire allocation—i.e. $\mathcal{T}'(y;\Lambda) = 0\ \forall y$. Consider a marginal increase in Λ. Then, the optimal t&T schedule becomes regressive: it features a positive lump-sum tax, and at all levels of income marginal rates are negative and/or average rates are falling in income.*

Proof. In Appendix A.3 the proof establishes that: When the overall tax payment is positive, average tax rates are always falling in income; and when the overall tax payment is negative, marginal rates are always negative. □

Away from the laissez-faire economy, which of these effects dominates is a quantitative question that we explore in detail next. Anticipating the results, we will find the distributional gains channel to remain quantitatively dominant: the optimal *t&T* system becomes less redistributive with rising living standards.

4 Quantitative Setup

We now move to the quantification of the effects of rising living standards on the optimal *t&T* system. For this purpose, we use two complementary approaches.

We start with a Ramsey approach and describe optimal parametric *t&T* systems in a rich dynamic incomplete-market setup. A dynamic model offers two main advantages. First, precautionary savings endogenously generate a distribution of expenditure given the observed distribution of income, which is crucial to disentangle efficiency from distribution concerns. Second, a model with savings generates dynamic moments such as MPCs and wealth effects, for which we have empirical counterparts, and which are intrinsically related to risk aversion as we derive in Section 4.3.3. Thus, a dynamic model is useful to discipline the DRRA property arising from NH preferences, alongside consumption composition.

We then use a Mirrlees approach in a static setup. This approach offers two advantages. First, it allows to check that the results are not driven by the specific *t&T* functional forms assumed in the Ramsey exercise. Second, it allows to build on the optimal tax formula in Lemma 2 and decompose the relative importance of the three channels of rising living standards identified in Section 3.

Section 4.1 formally introduces the dynamic model. Section 4.2 describes the calibration of preferences, growth, and changes in inequality in the dynamic model to the U.S. economy from 1950 to 2010. Section 4.3 checks the model implications of non-homotheticities on static decisions—that is, consumption and labor patterns—as well as on dynamic decisions—that is, wealth effects and MPCs. We further compare the implied level of DRRA in the model to empirical estimates provided in the literature. Finally, Section 4.4 addresses the calibration of the static model.

4.1 Dynamic Model: Setup

The dynamic model is a standard incomplete-market setup. Households are characterized by their productivity θ and holdings of a risk-free bond a. The household problem reads as follows:

$$\begin{aligned} V\left(a,\theta;\Lambda,p\right) &= \max_{e,a',n}\left\{u(e;\Lambda,p) - B\frac{n^{1+\varphi}}{1+\varphi} + \beta\mathbb{E}_{\theta'}\left[V\left(a',\theta';\Lambda,p\right)|\theta\right]\right\} \\ \text{s.t.}\quad & e + a' \leq \theta n + (1+r)a - \mathcal{T}\left(\theta n\right), \quad a' \geq 0, \end{aligned} \tag{6}$$

where the utility function u will be NH. Households discount the future with discount factor β and face a no-borrowing constraint. Productivity θ follows a stochastic process. The $t\&T$ system $\mathcal{T}(\cdot)$ is modeled as a parametric function of labor income. We describe all functional forms in the calibration section. The problem is cast in partial equilibrium, with the interest r and the vector of prices taken as exogenous.

4.2 Dynamic Model: Calibration

We calibrate the model in two points in time: 1950 and 2010. We use NH CES preferences with three sectors: agriculture/food, manufacturing/goods, and services. Rising living standards result from falling prices, disciplined by GDP per capita growth and changes in relative prices. Rising income inequality results from changes in the distribution of idiosyncratic productivity shocks. Taxes and transfers describe the U.S. fiscal system in 1950 and 2010. Table 1 presents all parameter values while Table 2 summarizes all targets.

4.2.1 Preferences

NH CES. The benchmark preference specification uses non-homothetic CES preferences. We rely on the estimates of Comin et al. (2021), based on micro data from the CEX, for the parameters ε_j, governing the expenditure elasticities of demand, and σ, governing the substitutability of the different commodities. We set $\sigma = 0.3$, $\varepsilon_A = 0.1$, $\varepsilon_G = 1.0$, and $\varepsilon_S =$

Table 1: Parameter Values

Preferences				
Discount factor	β	0.96	NH CES parameters	
Utility curvature	γ	0.77	σ	0.3
Frisch elasticity	$1/\varphi$	0.50	$(\varepsilon_A, \varepsilon_G, \varepsilon_S)$	$(0.10, 1.00, 1.80)$
Labor disutility	B	8.14	$(\Omega_A, \Omega_G, \Omega_S)$	$(0.05, 1.00, 11.03)$
Idiosyncratic Productivity			**Prices**	
Persistence	ρ_θ	0.9	r	0.02
Inequality	$\{\sigma_\theta, \alpha_\theta\}_{1950}$	$(0.27, 2.20)$	p^*_{1950}	$(3.11, 5.81, 1.84)$
Inequality	$\{\sigma_\theta, \alpha_\theta\}_{2010}$	$(0.30, 1.65)$	p^*_{2010}	$(1.00, 1.00, 1.00)$
Government				
Taxes	$\{\lambda, \tau\}_{1950}$	$(0.24, 0.11)$	$\{\lambda, \tau\}_{2010}$	$(0.27, 0.06)$
Spending	$\{T, G\}_{1950}$	$(0.01, 0.11)$	$\{T, G\}_{2010}$	$(0.02, 0.12)$

1.8. As such, agricultural products are the necessities, with a low expenditure elasticity of demand, whereas services are the luxury, with a high expenditure elasticity of demand. We set the parameters Ω_j of the NH CES to match aggregate sector shares in 2010, based on Herrendorf, Rogerson, and Valentinyi (2013): 8% for agriculture, 26% for goods, and 67% for services.[17]

Other preference parameters. We set the discount factor β to match a wealth-to-income ratio of 4.1 in 2010 (Piketty and Zucman 2014). We fix the Frisch elasticity at a standard value with $1/\varphi = 0.5$ and the labor disutility parameter B such that average labor supply in 2010 is 0.3. We target an average relative risk aversion in 2010 of 1, a standard value in the literature that often relies on log utility. With an implied curvature parameter γ equal to 0.77, this calibration delivers DRRA, as shown below in Figure 1. We discuss further the dynamics of risk aversion in Section 4.3.4, and provide a robustness check for γ in Section 5.3.

4.2.2 Growth and Prices

We fix the interest rate at 2% for both years. As is standard with this class of model, we can normalize the price vector in one period. We calibrate the three prices in the other period to match three moments: aggregate growth in GDP per capita from 1950 to 2010, and changing relative prices of agriculture and services to goods over the same time period.

Accounting for changes in relative prices is necessary to compare the model-implied rise in living standards, as captured by changes in sectoral expenditure shares, to its empirical counterpart which is measured in nominal terms. As discussed in Jaravel and Olivi (2024),

[17] We follow the final expenditure rather than value added approach in Herrendorf et al. (2013) since we are modeling household expenditure behavior rather than production.

Table 2: Targeted Data and Model Moments

Moment	Source	Data	Model
Moments related to Preferences, all 2010			
Agg. wealth/income	Piketty et al. (2014)	4.1	4.1
Avg. RRA	Standard value	1.00	1.00
Agg. shares: A,G (%)	Herrendorf et al. (2013)	$7.5, 25.6$	$7.5, 25.6$
Moments related to Prices			
Change p_a/p_g 1950-2010	Herrendorf et al. (2013)	1.87	1.87
Change p_s/p_g 1950-2010	Herrendorf et al. (2013)	3.16	3.16
GDP per capita growth	NIPA	3.34	3.34
Moments related to Inequality			
$\mathbb{V}[\log(y)]$ 1950, 2010	SCF+	$0.57, 0.78$	$0.57, 0.78$
Moments related to Government			
T/Y (%) 1950, 2010	NIPA	$1.2, 4.0$	$1.2, 4.0$
G/Y (%) 1950, 2010	NIPA, constant ratio	22.0	$22.0, 22.0$
ΔAMTR (%) 1950, 2010	Mertens et al. (2018)	$12.9, 8.7$	$12.9, 8.7$

changes in relative prices may also have efficiency and distributional implications on the optimal $t\&T$ system because heterogeneous households consume heterogeneous baskets of goods. We quantitatively isolate this force in a decomposition exercise in Section 5.2.

We compute aggregate growth in GDP per capita from 1950 to 2010 from National Income and Product Accounts (NIPA) to be equal to 3.3. We compute changes in relative prices based on Herrendorf et al. (2013). From 1950 to 2010, the relative price of agriculture (food) rises by a factor of 1.87 relative to goods, and the relative price of services rises by a factor of 3.16. These targets translate into falling prices for all commodities from 1950 to 2010, with the largest fall in goods and the smallest fall in services.

4.2.3 Inequality Dynamics

Household productivity follows an AR(1) process in logs, to which a Pareto tail is appended, with a time-varying Pareto tail parameter α_θ set to 2.2 in 1950 and 1.65 in 2010 (Aoki and Nirei 2017). We fix ρ_θ, the persistence of the productivity process, to 0.9 and set σ_θ the standard deviation of the innovation each period to match the variance of log income in 1950 (0.57) and 2010 (0.78) in the extended Survey of Consumer Finances (SCF+) of Kuhn, Schularick, and Steins (2020)—see Appendix B.1 for details on the SCF+ data.

The variance of log income is targeted explicitly, but the model provides a good fit for income inequality along the entire income distribution. As in the data, the income share of the bottom quintile falls by a third in the model, and the income share of the top quintile strongly increases—see Table C.2 in Appendix C.2.

4.2.4 Government

We restrict the analysis to a parametric but flexible functional form, following Ferriere, Grübener, Navarro, and Vardishvili (2023). The tax payment is given by

$$\mathcal{T}(y) = \exp\left[\log(\lambda)\left(y^{-2\tau}\right)\right] y - T. \tag{7}$$

The first part of the equation describes a two-parameter tax function, with parameter λ governing the level of taxes and parameter τ governing the progressivity, and T is a lump-sum transfer. As compared to the widely used loglinear tax function, popularized by Feldstein (1969) and Heathcote et al. (2017), it allows to better jointly match the bottom and the top of the tax distribution. Loosely speaking, T is disciplined by average tax-net-of-transfer rates and τ by the marginal tax rates at the top.

We set T and τ for the years 1950 and 2010 to match the transfer-to-output ratio and the difference in average marginal tax rates (AMTRs) between the top-10% and the bottom-90% of the income distribution. We measure transfers as: food stamps; Supplemental Security Income (SSI); refundable tax credits; unemployment insurance, workers' compensation and temporary disability insurance; various additional assistance programs; and Medicaid. Transfers amount to 1.2% of GDP in 1950 and 4.0% of GDP in 2010. We use data from Mertens and Montiel Olea (2018) to compute the difference in AMTRs, equal to 13% in 1950 and 9% in 2010.

Exogenous spending in 1950 is measured as total federal, state and local spending, deducting the transfers described above. We further deduct deficits from spending to match fiscal revenues over spending, to reach G/Y equal to 22.0%. We then assume spending to GDP to be fixed in 2010, to abstract from changes in fiscal pressure with effects on optimal taxes, as discussed in Heathcote and Tsujiyama (2021) and Ayaz, Fricke, Fuest, and Sachs (2023).[18] Finally, the parameter λ is determined by the restriction that the government budget has to clear period by period.[19]

4.3 Dynamic Model: Validation

We now validate the calibration by examining various additional moments. We investigate expenditure and labor supply patterns, both over time and in the cross-section. We also verify dynamic decisions with wealth effects and MPCs. We end this section with a comparison of

[18] Section 5.3 presents a robustness to account for rising fiscal pressure, with a calibration of tax revenues over GDP equal to 24.3% in 2010.

[19] See Appendix C.1 for further details on the calibration of taxes, transfers, and spending.

the implied degree of DRRA to estimates in the literature.

4.3.1 Expenditures

Aggregate expenditure shares over time. We investigate the change in aggregate sector shares between 1950 and 2010, to validate the rising living standards in the model. As shown in Table C.1 in Appendix C.2, the model captures well the structural change out of agriculture towards services, with an agricultural sector share of 17% (data: 22%), goods share of 49% (39%), and services share of 34% (39%) in 1950.

Expenditure inequality. We investigate the change in expenditure inequality between 1950 and 2010, to validate the change in distributional concerns in the model. Expenditure inequality in the model is the result of income inequality, which we match, and private savings decisions.

In line with evidence, the expenditure distribution is more equal than the income distribution. In 2010, the variance of log expenditure in the model is 0.42, close to the number of 0.36 reported in Attanasio and Pistaferri (2014) using CEX data. The discrepancy traces back to our assumption of a Pareto tail in the income distribution, while the CEX does not oversample high-income households. The model also matches well the distribution of wealth by quintile, as reported in Table C.2.

There is no evidence on the distribution of expenditure in 1950. Yet, the model generates a reasonable wealth-to-income ratio (Table C.1) and distribution of wealth by quintile (Table C.2), which is informative of the capacity of the model to also generate a reasonable distribution of log expenditure. We obtain a variance of log expenditure of 0.33 in 1950 in the model, thus smaller than in 2010.[20]

Expenditure shares in the cross-section. We investigate cross-sectional heterogeneity in expenditure sector shares in 2010 in the model to validate further the preference parameters $\{\varepsilon_j\}$ estimated in Comin et al. (2021). The agriculture expenditure share is 8.8 percentage points larger in the bottom than in the top expenditure quintile in 2017 (Meyer and Sullivan 2023), to be compared to a 11.1 percentage points difference in the model. Instead, the services expenditure share is 10 percentage points smaller in the bottom than in the top income quintile in 2010 (Boppart 2014), to be compared to 11.1 percentage points

[20]There is no consensus in the literature on how much consumption inequality has increased over time. In particular, Krueger and Perri (2006) or Heathcote, Perri, Violante, and Zhang (2023), among others, report stable or moderately increasing consumption inequality. Accounting for measurement error, Attanasio, Hurst, and Pistaferri (2014) and Aguiar and Bils (2015) find instead that consumption inequality has increased as much as income inequality since the 1980s. Meyer and Sullivan (2023) focus on well-measured consumption only in the Consumer Expenditure Survey and find that consumption inequality rose less than income inequality between 1961 and 2017. See this paper also for a more complete review of the different approaches and results in the literature.

in the model. Overall, the model captures well cross-sectional heterogeneity in expenditure sector shares in 2010. Again, there is no cross-sectional evidence for 1950.

4.3.2 Labor Supply

Recent literature has documented key patterns of labor supply over time, across countries, and in the cross-section within a country. Boppart and Krusell (2020) find a steady fall in hours worked by roughly 0.5% per year as a robust pattern of labor supply over time for different countries. For the postwar United States, McGrattan and Rogerson (2004) and Ramey and Francis (2009) find a fall in hours per worker of 5-7%.[21] Cross-sectional patterns of labor supply have also changed over time. Before the 1970s, low-wage workers worked more hours than high-wage workers, a pattern which has reversed since then—see Costa (2000), Heathcote, Perri, and Violante (2010), Mantovani (2023), and Heathcote et al. (2023).

We compute aggregate and cross-sectional changes in labor supply in the model. Aggregate labor supply falls by 7% over time, a number which is somewhat high but consistent with the estimates in the literature. The correlation between hours worked and hourly wage increases by 10 points from 1950 to 2010—Heathcote et al. (2023) compute an increase of 22 points for men and 7 points for women from 1967 to 2021. A larger value of the curvature parameter γ would increase the change in the correlation, at the expense of a larger fall in labor supply. Overall, this parsimonious model captures fairly well the effects of growth on labor supply, both in the aggregate and the cross-section.

4.3.3 Wealth Effects and MPCs

We exploit the dynamic dimension in the model to further validate the calibration of preferences and the implied degree of DRRA. In particular, we link RRA to concepts that are better measurable in the data, such as MPCs and wealth effects. To do so, we derive the following expression from the households' budget constraint and savings decisions, where η denotes the wealth effect (see Appendix C.3):

$$\eta\left(\varphi\frac{e}{\theta n}+\frac{e\mathcal{T}''\left(\theta n\right)}{\mathcal{T}'\left(\theta n\right)}\right)=\text{MPC}\times\text{RRA}. \tag{8}$$

In 2010, the calibrated model produces MPCs and wealth effects well in line with available evidence. For MPCs, we compute the expenditure response to a \$500 increase in wealth, yielding an 17% average. While this is relatively low compared to most available evidence, we consider it a success for this class of models with only one asset calibrated to the entire stock

[21] In terms of total hours this is compensated by rising female labor force participation, a pattern we abstract from in the model.

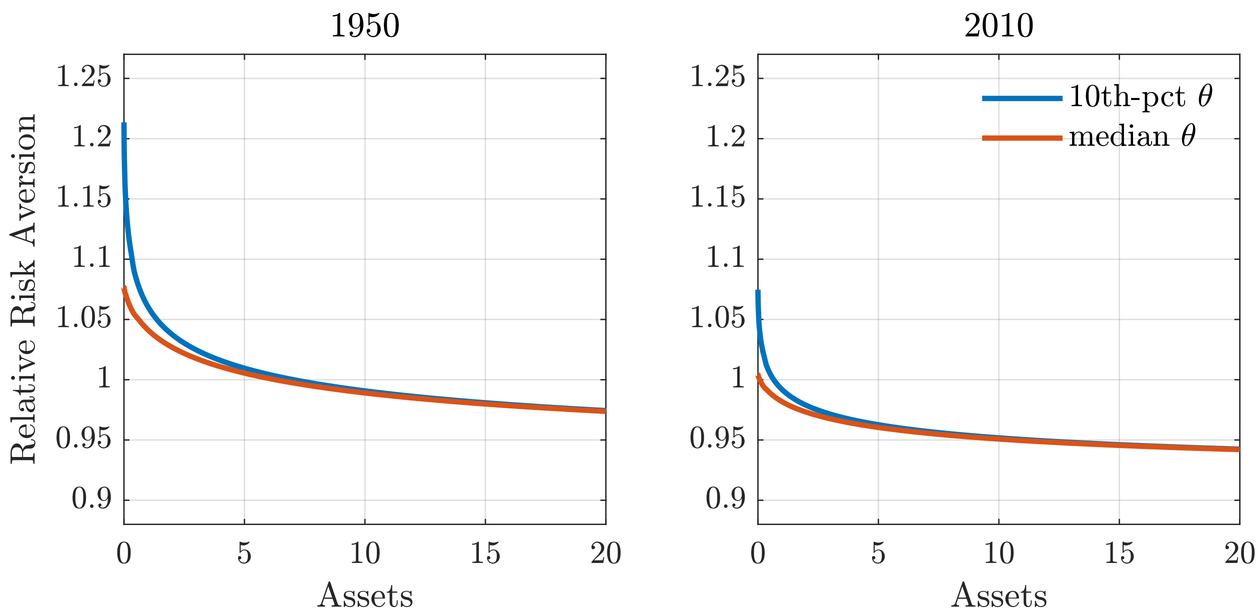

Figure 1: Relative Risk Aversion

Notes: Figure 1 plots dispersion in relative risk aversion in the calibrated model in 1950 (left panel) and 2010 (right panel). Wealth is normalized by mean wealth.

of wealth (Kaplan and Violante 2022). For wealth effects, we compare the model response to a one-time unanticipated wealth shock to the evidence of Golosov, Graber, Mogstad, and Novgorodsky (2024), who measure the earnings response to lottery winnings using the universe of U.S. taxpayers. The model replicates closely the observed earnings fall of $2.3 in response to a $100 wealth shock.

4.3.4 DRRA

Finally, we directly compute the model implied degree of DRRA and compare it to the evidence discussed in Section 2.3. Figure 1 reports relative risk aversion by wealth in 1950 and 2010 for different levels of θ. Cross-sectional dispersion in risk aversion is small, both in 1950 and in 2010. The average relative risk aversion, which is calibrated to 1 in 2010, is only equal to 1.07 in 1950. Atkeson and Ogaki (1996) find that the IES of the richest households in India is 60% higher than the one of the poorest households. The ratio of the IES between the U.S. and India is roughly 1.5. In the U.S. time series they estimate an increase in the IES from 0.38 to 0.41 from 1929 to 1988. Blundell et al. (1994) report variation in the IES from the 10th to the 90th percentile of UK households ranging from 0.66 to 1.10 or 0.96 to 2.8, depending on the specification. Overall, the calibrated model implies a modest degree of DRRA, well within the range of plausible estimates—that is, the calibration is conservative.

4.4 Static Model: Calibration

Finally, we briefly describe the quantification of the static model. We partly use the dynamic model to quantitatively discipline the static model, as we explain next. Table C.3 summarizes all parameters.

We calibrate preferences, growth and prices, and government parameters as in the dynamic model. In contrast to the dynamic model, there is no distinction between after-tax income and expenditure in this environment. For the calibration of this static model, we therefore follow a partial insurance approach—that is, we calibrate productivities such that the model is consistent with expenditure inequality in the data, as expenditures determine dispersion in marginal utilities.

Specifically, we calibrate the skill distribution as an exponentially modified Gaussian distribution (EMG), as in Heathcote and Tsujiyama (2021). For 2010, we set the Pareto tail parameter to 3.3 (Gaillard, Hellwig, Wangner, and Werquin 2023; Toda and Walsh 2015). We then set the variance of the normal shock in 2010 to match a variance of log expenditure of 0.36 (Attanasio and Pistaferri 2014). In 1950, there is no data on the expenditure distribution. We assume that, as in 2010, the consumption tail parameter is twice as large as the income tail parameter. This strategy implies a consumption parameter of 4.4 in 1950.[22] Similarly, we assume that, as in 2010, the variance of log expenditure is about 40% of the variance of log income. This strategy implies a variance of log expenditure of 0.26 in 1950.[23] Appendix C.4 provides more details on computations and calibration and summarizes all parameters.

We validate this calibration by examining labor supply behavior over time and in the cross-section. Over time, aggregate labor supply falls by 5%. Labor supply is monotonically decreasing in productivity in 1950 and monotonically increasing in productivity in 2010, as in the data. Average risk aversion amounts to numbers very comparable to the dynamic model, at 1.07 in 1950 and 1.00 in 2010.

5 Optimal Policy: Quantification

We now quantify the effects on the optimal $t\&T$ system of the rising living standards relative to the rising inequality. Section 5.1 follows a Ramsey approach in the dynamic model and computes the optimal fiscal system in 2010 within the class of $t\&T$ functions described in Section 4.2. Section 5.2 complements the analysis with the optimal fully nonlinear $t\&T$ system in the static model, and further uses the theoretical results from Section 3 to decompose

[22] The tail parameter for the income distribution is measured at 2.2 in 1950 in Aoki and Nirei (2017).

[23] Details on the income data for 1950 are provided in Section B.1.

the effects of rising living standards. Section 5.3 presents various robustness exercises.

5.1 Ramsey Analysis in Dynamic Model

We start with the Ramsey analysis in the dynamic model. We proceed in three steps. First, we find inverse optimum Pareto weights that make the observed $t\&T$ system in 1950 optimal. Second, fixing Pareto weights, we compute the optimal $t\&T$ system in 2010 when only accounting for rising inequality. Third, we also account for rising living standards through the fall in prices.

Pareto Weights. We start from the 1950 calibrated $t\&T$ system and find the Pareto weights under which it is optimal. When evaluating the optimal $t\&T$ system in 2010, we then assume fixed social preferences over time.

Heterogeneity is two-dimensional in the dynamic model, with households differing both in productivity and wealth. A one-dimensional measure, capturing how well-off a household is, is expenditure.[24] The $t\&T$ system in 1950 is characterized by two parameters, T and τ. Hence, we use a two-parameter function for the Pareto weights w, which we assume of the following form: $w\left(\pi_i(e_i)\right) = \exp\left(\mu\pi_i(e_i) + \nu\pi_i(e_i)^2\right)$. This functional form, also used in Le Grand, Ragot, and Rodrigues (2024), flexibly extends the one-dimension functional forms used in the literature (Chang et al. 2018; Heathcote and Tsujiyama 2021). Pareto weights increasing at different rates at different levels of expenditure e_i allow to jointly match redistribution at the top and at the bottom of the distribution. Importantly, the Pareto weight depends on the percentile π_i of the expenditure distribution, rather than on the expenditure level itself. Defining weights as a function of percentiles rather than levels is irrelevant for 1950, but important for 2010. Indeed, the increasing Pareto weights that match the observed $t\&T$ system in 1950 at the top of the expenditure distribution would imply a mechanical increase in the weights on the rich when accounting for rising inequality—see Appendix C.5 for more details.

1950. Figure 2 reports the calibrated (and optimal) marginal and average $t\&T$ rates in 1950. Average rates are only very modestly negative at the bottom, given the small transfer at 1.2% of GDP. A comprehensive measure of redistribution is the difference between the average $t\&T$ rates of the top and bottom labor income deciles. We denote this measure $\mathcal{R}$ and use it as our reference measure to evaluate the overall level of redistribution at a given point in time. In 1950, redistribution is moderate: $\mathcal{R}$ amounts to 24 p.p.—that is, the top-10 average $t\&T$ rate is only 24 p.p. higher than the bottom-10 rate.

[24] Chang, Chang, and Kim (2018) also use an inverse optimum approach conditioning Pareto weights on expenditures. In Section C.6, we consider an alternative formulation of Pareto weights as a function of productivity and obtain similar results.

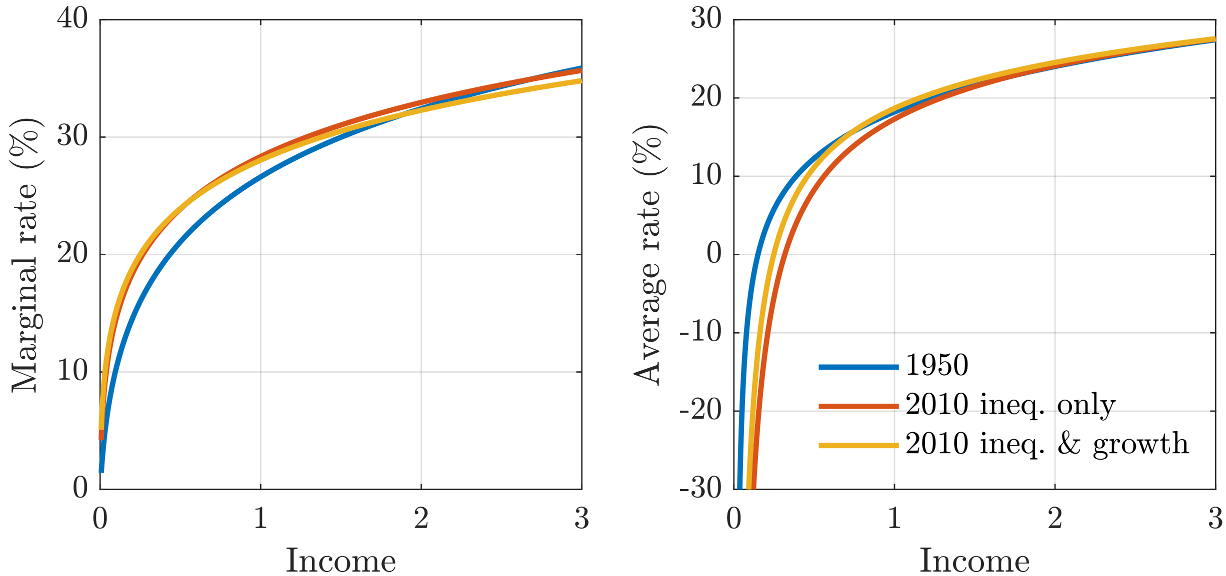

Figure 2: Optimal $t\&T$ Rates in the Dynamic Model

Notes: Figure 2 shows the optimal marginal and average $t\&T$ rates for: (1) the 1950 inverse optimum; (2) "2010 ineq. only", with only the rise in inequality from 1950 to 2010; (3) "2010 ineq. & living std.", with rising inequality and falling prices. Income is normalized by mean income.

2010: Rising inequality. Starting from the 1950 economy, we first adjust only inequality to 2010 levels and compute the optimal $t\&T$ system. To do so, we modify both the Pareto tail parameter α and the variance of the innovation of the AR(1) process governing the dispersion in productivity, but keep prices constant at their 1950 level.

As shown in Figure 2, the $t\&T$ system becomes more redistributive when inequality rises. Taxes become more progressive, as marginal tax rates rise across most of the income distribution and especially so at the top. The government raises more revenues and redistributes through a larger lump-sum transfer, amounting to 4.7% of output. Overall, the 2010 optimal $t\&T$ system provides much more redistribution, with $\mathcal{R}$ increasing to 53 p.p. This result echoes the typical finding in the literature that rising inequality calls for more redistribution.

2010: Rising inequality and rising living standards. The third scenario in Figure 2 accounts for rising living standards, in addition to rising inequality. To do so, we adjust prices to their 2010 level.

When also accounting for rising living standards, marginal tax rates do increase, as compared to the 1950 $t\&T$ system, but not as much as with rising inequality only. The optimal lump-sum transfer amounts to only 3.7% of output instead of 4.7%. Overall, redistribution is higher than in 1950, but not as high as with rising inequality only: $\mathcal{R}$ rises to 45 p.p. instead of 53 p.p. That is, rising living standards dampen by 30% the desired increase in redistribution due to rising inequality.

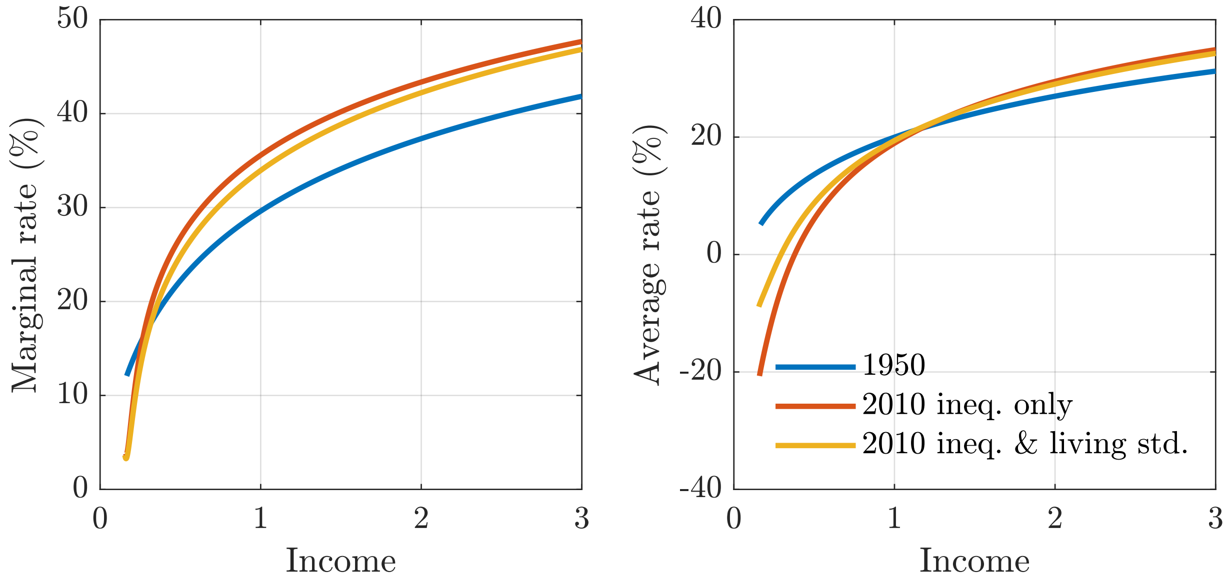

Figure 3: Optimal $t\&T$ Rates in the Static Model

Notes: Figure 3 shows the optimal marginal and average $t\&T$ rates in the Mirrlees setup for: (1) the 1950 inverse optimum; (2) "2010 ineq. only", with only the rise in inequality from 1950 to 2010; (3) "2010 ineq. & living std.", with rising inequality and living standards. Income is normalized by mean income.

Interestingly, the optimal $t\&T$ system is roughly comparable to the calibrated one in 2010. In the 2010 calibration, $\mathcal{R}$ equates 45 p.p. and the lump-sum transfer amounts to 4.0%.

5.2 Mirrlees Analysis in Static Model

We now turn to the optimal policy analysis in the static Mirrlees model with unrestricted nonlinear income taxes. We follow the same approach as with the dynamic model. We start with finding inverse optimum weights making the 1950 $t\&T$ system optimal. In this framework, we can find a unique set of Pareto weights as a function of productivity. Without savings, productivity fully captures inequality in earnings potential (Bourguignon and Spadaro 2012). As before, we keep these weights constant over time as functions of the position in the distribution.

Figure 3 shows the optimal marginal and average $t\&T$ rates in the static model for the three cases: (1) the 1950 calibrated $t\&T$ function; (2) the optimal 2010 $t\&T$ system with rising inequality only; and (3) the optimal 2010 $t\&T$ system when also accounting for rising living standards. Results are comparable to those in the dynamic model. Redistribution $\mathcal{R}$ is equal to 19 p.p. in the calibration of the 1950 economy; $\mathcal{R}$ rises to 42 p.p. with rising inequality only, but only to 35 p.p. when also accounting for rising living standards. That is, rising living standards dampen by 32% the desired increase in redistribution $\mathcal{R}$ due to rising inequality. In line with this finding, the transfer-to-output ratio, which equates 1.2%

in the calibration of the 1950 economy, rises to 3.9% with rising inequality only, but only to 1.9% when also accounting for rising living standards.

Overall, we consistently find across both Ramsey and Mirrlees approaches that rising living standards moderate the optimal increase in redistribution due to rising inequality.

We now use the quantified version of the static model for two decompositions.

Decomposition of rising living standards: three channels. The first decomposition builds on the optimal tax formula in Lemma 2 and the comparative statics in Propositions 4-6 to quantify the main drivers of the effect of rising living standards on optimal taxes. Recall that changes in Λ affect the tax formula through three channels: the *distributional gains* channel (Proposition 4), the *efficiency costs* channel (Proposition 5), and the *income distribution* channel (Proposition 6). From the 1950 calibrated $t\&T$ system, we first account for rising inequality only. Then, we consider rising living standards adding each of the three channels sequentially. Optimal $t\&T$ rates are plotted in the top left panel of Figure 4.

We start from the "Inequality only" $t\&T$ system, which is computed with the distribution of types θ as in 2010 but with prices from 1950. We first isolate the *distributional gains* channel by computing marginal utilities under 2010 prices, while still using 1950 prices to compute income effects and hours worked. As the dispersion in marginal utilities declines, redistribution decreases significantly, with marginal rates falling by 2 to 4 p.p. across the board. Overall, the *distributional gains* channel has large effects, overshooting the total effect of rising living standards on redistribution: It dampens the desired increase in $\mathcal{R}$ by 41%, compared to 32% in total.

We then also account for the *efficiency costs* channel by computing income effects under 2010 prices, while still computing hours worked given 1950 prices. Theoretically, larger income effects have ambiguous effects. Quantitatively, the opposing effects essentially cancel out. Marginal tax rates and the optimal redistribution $\mathcal{R}$ barely change relative to the previous scenario.

Finally, we account for the *income distribution* channel by also computing optimal hours worked under 2010 prices. This last step exactly retrieves the 2010 optimal $t\&T$ system—that is, it reduces the dampening of redistribution from 41% to 32%. The *income distribution* channel partially reverses the *distribution gains* channel as, for a constant skill inequality, income inequality is higher under 2010 prices than under 1950 prices: At the optimum, the variance of log expenditure increases moderately, from 0.29 to 0.30, calling for more redistribution.

Overall, the first effect dominates quantitatively: rising living standards decrease optimal redistribution mostly due to lower distributional gains.

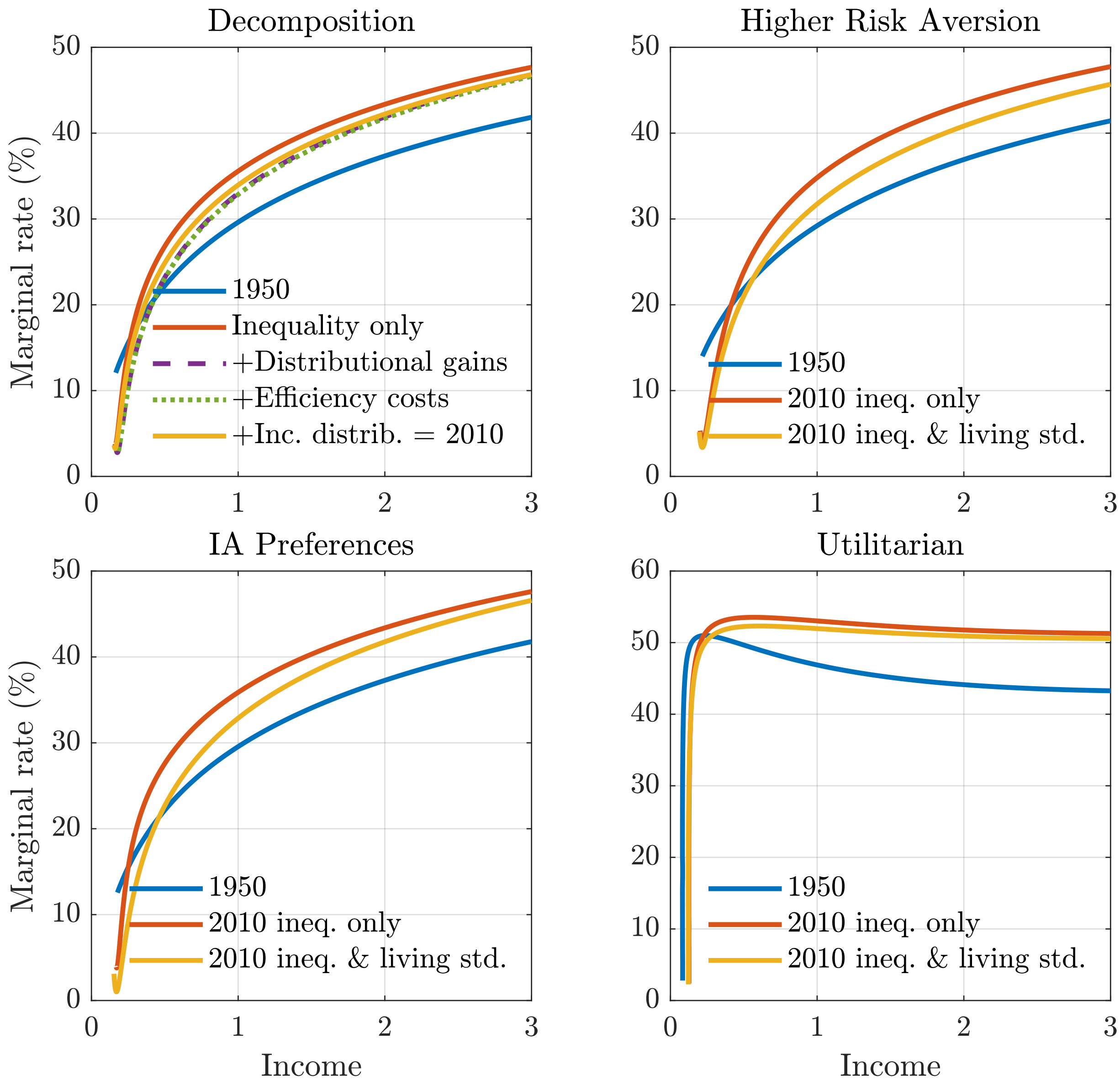

Figure 4: Decompositions and Robustness in the Static Model

Notes: The top left panel presents a three-step decomposition from the 2010 "inequality only" case to the 2010 optimal $t\&T$ system with rising inequality and rising living standards, into: *distributional gains*, *efficiency costs*, and *income distribution* channels. The three other panels present robustness exercises: with higher relative risk aversion (top right panel), with IA preferences (bottom left panel), and with a Utilitarian planner (bottom right panel). Income is normalized by mean income.

Relative prices. The second decomposition disentangles the effect of the aggregate fall in prices, driven by changes in level Λ, from the effect of changes in relative prices.

Starting from the "Inequality only" case, we isolate the effect of the aggregate fall in prices. To do so, we compute a counterfactual where relative prices in 2010 remain as in 1950 but all prices fall homogeneously to generate the same change in GDP per capita as in the data. We compare the optimal $t\&T$ system to the 2010 optimum which also accounts for changes in relative prices.

The increase in redistribution $\mathcal{R}$ is dampened by only 22% with a homogeneous fall in prices, compared to 32% in the benchmark. That is, changes in relative prices account for roughly a third of the total effect of rising living standards. This is because the larger fall in the price of necessities from 1950 to 2010 raised living standards especially for those at the bottom of the distribution. Overall, the effects of rising living standards primarily stem from the homogeneous price fall.

5.3 Robustness

To conclude the analysis, we conduct four robustness exercises. The first two exercises explore alternative preference calibrations. First, we recalibrate the economy to a larger degree of RRA. Second, we replicate the benchmark exercise using the other NH preferences generally used in the literature, the IA preferences of Alder et al. (2022). The third exercise explores an alternative welfare function. The last exercise considers increasing fiscal pressure, which also influences how redistribution should evolve over time. We conduct these exercises in the quantitatively more tractable Mirrlees environment.

Higher risk aversion. We calibrate an alternative economy with higher risk aversion, with the curvature parameter γ moving from 0.77 to 1.5—see Appendix C.4 for details. Optimal $t\&T$ rates are plotted in the top right panel of Figure 4.

Average risk aversion now amounts to 1.36 in 2010, and to 1.58 in 1950 as higher levels of risk aversion also amplify DRRA. This increase in the level of risk aversion magnifies significantly the effects of living standards on the optimal $t\&T$ system. The optimal increase in redistribution $\mathcal{R}$ is dampened by more than 70%, compared to 32% in the benchmark. From 1.2% of GDP in 1950, the optimal transfers increase to 5.2% in 2010 with rising inequality only, but fall to -0.3% when also accounting for rising living standards. Yet, this alternative calibration with higher risk aversion and larger effects of rising living standards also generates a counterfactually large fall in aggregate labor supply, of about 20%.

IA preferences. We perform the analysis replacing the NH CES preferences with the other state-of-the-art NH preferences, the IA preferences of Alder et al. (2022) introduced in Section 3. We use their functional form for the $\mathbf{D}$ term:

$$\mathbf{D}(p^*) = \frac{\nu(1-\iota)}{\eta}\left(\left[\frac{\tilde{D}(p^*)}{B(p^*)}\right]^{\eta} - 1\right), \ \tilde{D}(p^*) = \left(\sum_{j\in J} \theta_j {p_j^*}^{1-\xi}\right)^{\frac{1}{1-\xi}},$$

with $\nu \geq 0$, $\eta \in (0,1)$, $\xi > 0$, $\sum_{j\in J} \theta_j = 1$, and $\theta_j \geq 0\ \forall j$. We calibrate these preferences to match the same targets as with the NH CES.[25]

Key to the calibration are the levels of $\{\bar{c}_j\}$, which govern the sign of the generalized Stone-Geary term $\mathbf{A}$. As explained in Lemma 2, $\mathbf{A} > 0$ is a necessary condition for the IA preferences to generate a fall in labor supply, and a sufficient condition to satisfy DRRA. The obtained fall in aggregate labor supply is small, at -0.1%, yet generating a stronger DRRA pattern than in the benchmark case. Average risk aversion equals 1.08 in 1950, as with the NH CES preferences, but amounts to 1.55 for the poorest in 1950, to be compared to 1.23 with the NH CES preferences. Aggregate risk aversion falls to 0.94 in 2010, to be compared to 1.0 with the NH CES preferences.

Therefore, the effects of rising living standards are larger than in the NH CES benchmark, as shown in the bottom left panel of Figure 4. The optimal increase in redistribution $\mathcal{R}$ is dampened by 44%, compared to 32% in the benchmark. The transfer-to-output ratio moves from 1.2% in 1950 to 3.1% with rising inequality only, and to 0.4% when also accounting for rising living standards. Overall, rising living standards reduce the increase in the optimal level of redistribution due to rising inequality, and this result holds regardless of the exact functional form of NH preferences used.

Utilitarian planner. Optimal $t\&T$ rates under a Utilitarian planner are plotted in the bottom right panel of Figure 4. Marginal rates are much higher across all scenarios, a common finding in the literature (Heathcote and Tsujiyama 2021; Saez 2001). In 1950, the optimal transfer amounts to 19.2% of GDP. With rising inequality only, optimal marginal rates increase and the transfer reaches 22.2% of GDP. Adding rising living standards, optimal marginal rates increase by less and finance a lower transfer, at only 20.6% of GDP. The optimal increase in redistribution $\mathcal{R}$ is dampened by about 20%. Overall, the effects of living standards are somewhat weaker under the Utilitarian planner, as the optimal $t\&T$ system is closer to the Laffer bound, but they remain quantitatively significant.

Fiscal pressure. The benchmark assumed constant spending between 1950 and 2010, at the measured 1950 level of 22%. Yet, the level of fiscal pressure matters to the optimal progressivity, as shown in Heathcote et al. (2017) and Heathcote and Tsujiyama (2021): higher fiscal pressure decreases optimal redistribution. This robustness exercise adjusts the level of spending in 2010 to match a level of fiscal revenues—that is, of spending net of deficits—of 24.3%, as reported in NIPA.

With rising fiscal pressure, the optimal transfer-to-output ratio goes up from 1.2% in 1950 to only 3.1% with rising inequality only, and decreases to 1.1% when also accounting

[25] The parameters for the IA preferences are the following: $\iota = \eta = 0.1$, $\bar{c}_A = 0.025$, $\bar{c}_G = 0.00$, $\bar{c}_S = 0.003$, $\sigma = 0.001$, $\Omega_A = 0.05$, $\Omega_G = 0.42$, $\nu = 15$, $\xi = 2$, $\theta_A = 0.24$, $\theta_G = 0.625$.. See Table C.3 for the calibration of all other parameters.

for rising living standards. That is, as expected, higher fiscal pressure generally reduces the optimal increase in redistribution between 1950 and 2010. Yet, the effect of living standards is unchanged. The optimal transfer-to-output ratio is reduced by 2 p.p., similar to the benchmark; and the optimal increase in redistribution $\mathcal{R}$ is dampened by 35%, also comparable to the benchmark.

Overall, rising living standards quantitatively matter to the optimal $t\&T$ system, and this result holds across various assumptions regarding preferences, Pareto weights, and the government budget constraint.

6 Conclusion

This paper explored the impact of rising living standards on the optimal design of the $t\&T$ system. With NH preferences, rising living standards weaken distributional concerns while having ambiguous effects on efficiency concerns. Quantifying these forces, we found that rising living standards, an increase in the first moment of the income distribution, significantly dampen the optimal increase in redistribution due to rising inequality, a heavily-scrutinized change in the second moment.

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A Theory

A.1 Heterogenous Expenditure Elasticities

A.1.1 NH CES Preferences

We abuse notation for the following proofs and use $\mathcal{C}(e) = \mathcal{C}(e; \Lambda, p)$.

Risk aversion. Differentiating the expenditure function (2), denoting $\Omega_j^* \equiv p_j^{*1-\sigma}\Omega_j = (p_j/\Lambda)^{1-\sigma}\,\Omega_j$, one obtains

$$\mathcal{C}_e(e) = (1-\sigma)e^{-\sigma}\left(\sum_j \Omega_j^* \varepsilon_j \mathcal{C}(e)^{\varepsilon_j - 1}\right)^{-1} \tag{9}$$

$$\mathcal{C}_{ee}(e) = -\frac{\mathcal{C}_e(e)}{e}\left(\sigma + \mathcal{C}_e(e)e\frac{\sum_j \Omega_j^*\varepsilon_j(\varepsilon_j - 1)\mathcal{C}(e)^{\varepsilon_j-2}}{\sum_j \Omega_j^*\varepsilon_j\mathcal{C}(e)^{\varepsilon_j-1}}\right)$$

where $\mathcal{C}_e(e) > 0\ \forall e$. Rearranging this yields risk aversion equal to

$$\gamma(e) = \sigma + (1-\sigma)\frac{1}{\chi(\mathcal{C}(e))}\left(\gamma - 1 + \zeta(\mathcal{C}(e))\right),$$

$$\text{where} \quad \chi(C) \equiv \frac{\sum_j \Omega_j^*\varepsilon_j C^{\varepsilon_j}}{\sum_j \Omega_j^* C^{\varepsilon_j}} \quad \text{and} \quad \zeta(C) \equiv \frac{\sum_j \Omega_j^*\varepsilon_j^2 C^{\varepsilon_j}}{\sum_j \Omega_j^*\varepsilon_j C^{\varepsilon_j}}.$$

Proof of Lemma 1.

Proof. Using (9), one can rewrite the first term in (4) as

$$\gamma\frac{\mathcal{C}_e(e)e}{\mathcal{C}(e)} = \gamma(1-\sigma)\frac{\sum_j \Omega_j^*\mathcal{C}(e)^{\varepsilon_j}}{\sum_j \Omega_j^*\varepsilon_j\mathcal{C}(e)^{\varepsilon_j}}. \tag{10}$$

As $\mathcal{C}_e(e) > 0$ and $\gamma(1-\sigma) > 0$, we only need to show that the fraction in (10) is decreasing in C, i.e. that the inverse of the fraction in (10) is increasing in C:

$$\frac{\partial}{\partial C}\left[\frac{\sum_j \Omega_j^*\varepsilon_j C^{\varepsilon_j}}{\sum_j \Omega_j^* C^{\varepsilon_j}}\right] = \left(\sum_j \Omega_j^* C^{\varepsilon_j}\right)^{-2}\frac{1}{C}\frac{1}{2}\sum_k\sum_j \Omega_k^*\Omega_j^*(\varepsilon_k - \varepsilon_j)^2 C^{\varepsilon_k+\varepsilon_j} > 0,$$

which completes the proof. □

Proof of Proposition 1.

Proof. Bohr et al. (2023) define $\hat{\varepsilon}_j \equiv (1-\sigma)\varepsilon_j$ and make the following set of assumptions:

1. The price parameters $\{p_i^*\}_{i\in[0,1]}$ and taste parameters $\{\Omega_i\}_{i\in[0,1]}$ have a log-linear relationship with $\{\hat{\varepsilon}_i\}_{i\in[0,1]}$, with a regularity condition regarding the intercept.

2. $\{\hat{\varepsilon}_i\}_{i\in[0,1]}$ follow a gamma distribution: $\hat{\varepsilon}_i \sim \Gamma(\alpha, \beta)$, with $\alpha > 0$ and $\beta > 0$.

Then, they obtain a closed-form relationship between e and $\mathcal{C}(e)$ as shown in equation (7) of their paper:

$$\log\mathcal{C}(e) = \hat{Y} - \frac{\Psi}{1-\sigma}e^{-\frac{1-\sigma}{\alpha}},$$

where $\hat{Y} \in \mathbb{R}$ and $\Psi \in \mathbb{R}_+$. As such, we obtain the following closed forms for the derivatives:

$$\mathcal{C}_e(e) = \frac{\Psi}{\alpha} e^{-\frac{1-\sigma}{\alpha}-1} \mathcal{C}(e), \quad \mathcal{C}_{ee}(e) = \frac{\Psi}{\alpha} e^{-\frac{1-\sigma}{\alpha}-1} \mathcal{C}_e(e) - \left(\frac{1-\sigma}{\alpha} + 1\right) \frac{\mathcal{C}_e(e)}{e}.$$

Thus, recalling (4), we can express risk-aversion as:

$$\begin{aligned} \gamma(e) &= \underbrace{\gamma \times \frac{\Psi}{\alpha} e^{-\frac{1-\sigma}{\alpha}}}_{\text{first term}} + \underbrace{-\frac{\Psi}{\alpha} e^{-\frac{1-\sigma}{\alpha}} + \left(\frac{1-\sigma}{\alpha} + 1\right)}_{\text{second term}} \\ &= (\gamma - 1)\frac{\Psi}{\alpha} e^{-\frac{1-\sigma}{\alpha}} + \left(\frac{1-\sigma}{\alpha} + 1\right). \end{aligned} \tag{11}$$

As $\sigma < 1$, it follows that $\gamma'(e) < 0$ iff $\gamma > 1$. □

Proof of Corollary 1.

Proof. Using equation (2), the household's maximization problem is given by:

$$\max_n \frac{\mathcal{C}(e; \Lambda, \overline{p})^{1-\gamma}}{1-\gamma} - B\frac{n^{1+\varphi}}{1+\varphi} \text{ s.t. } e = n.$$

The first-order condition reads:

$$\mathcal{C}(e; \Lambda, \overline{p})^{-\gamma} \mathcal{C}_e(e; \Lambda, \overline{p}) = Be^{\varphi}.$$

Linearity of the expenditure function in Λ implies that:

$$\begin{aligned} \mathcal{C}_\Lambda(e; \Lambda, \overline{p}) &= \mathcal{C}_e(e; \Lambda, \overline{p})\frac{e}{\Lambda} \\ \mathcal{C}_{\Lambda e}(e; \Lambda, \overline{p}) &= \mathcal{C}_{ee}(e; \Lambda, \overline{p})\frac{e}{\Lambda} + \mathcal{C}_e(e; \Lambda, \overline{p})\frac{1}{\Lambda} \end{aligned}$$

Derivating both sides with respect to Λ, we obtain:

$$\begin{aligned} -\gamma \mathcal{C}(e; \Lambda, \overline{p})^{-\gamma-1} \left(\mathcal{C}_\Lambda(e; \Lambda, \overline{p}) + \mathcal{C}_e(e; \Lambda, \overline{p}) e_\Lambda\right) \mathcal{C}_e(e; \Lambda, \overline{p}) + \ldots \\ \ldots \mathcal{C}(e; \Lambda, \overline{p})^{-\gamma} \left(\mathcal{C}_{ee}(e; \Lambda, \overline{p}) e_\Lambda + \mathcal{C}_{\Lambda e}(e; \Lambda, \overline{p})\right) = B\varphi e^{\varphi-1} \\ \frac{1}{\Lambda} - \left(e_\Lambda + \frac{e}{\Lambda}\right) \frac{1}{e} \left(\gamma \frac{\mathcal{C}_e(e; \Lambda, \overline{p}) e}{\mathcal{C}(e; \Lambda, \overline{p})} - \frac{\mathcal{C}_{ee}(e; \Lambda, \overline{p}) e}{\mathcal{C}_e(e; \Lambda, \overline{p})}\right) = \varphi \frac{e_\Lambda}{e} \\ \frac{1}{\Lambda}(1 - \gamma(e; \Lambda, \overline{p})) = \frac{e_\Lambda}{e}(\varphi + \gamma(e; \Lambda, \overline{p})). \end{aligned}$$

Using the budget constraint that $e = n$ delivers that $n_\Lambda / n < 0 \Leftrightarrow \gamma(e; \Lambda, \overline{p}) > 1$. Using equation (11) delivers $\gamma > 1$ as a necessary and sufficient condition for $\gamma(e; \Lambda, \overline{p}) > 1$. □

Rescaling. Comin et al. (2021) show that all ε_j can be multiplied by a positive scalar without implications on intratemporal consumption allocations. We show next that the rescaling irrelevance extends to (i) risk aversion, (ii) labor supply, when γ and B are rescaled appropriately.

When multiplying all ε_j by a scalar ι, one needs to rescale $1-\gamma$ by that same scalar—that is, $\gamma_\iota \equiv 1-\iota(1-\gamma)$, where x_ι defines the rescaled version of variable x. The expenditure function defines a new $\mathcal{C}_\iota(e)$ which appears in both numerators and denominators of χ and ζ. Thus, we have $\chi_\iota = \iota\chi$ and $\zeta_\iota = \iota\zeta$, and risk aversion becomes

$$\begin{aligned}\gamma_\iota(e) &= \sigma + (1-\sigma)\frac{1}{\iota\chi(\mathcal{C}(e))}\left(\gamma_\iota - 1 + \iota\zeta(\mathcal{C}(e))\right)\\ &= \sigma + (1-\sigma)\frac{1}{\iota\chi(\mathcal{C}(e))}\left(\iota(\gamma-1) + \iota\zeta(\mathcal{C}(e))\right) = \gamma(e).\end{aligned}$$

Rescaling the curvature parameter γ as defined above leaves risk aversion unchanged.

We turn to labor supply. When multiplying all ε_j by a scalar ι, one needs to rescale the labor disutility parameter such that $B_\iota \equiv B/\iota$. Abstracting from taxes w.l.o.g. the first-order condition reads

$$\mathcal{C}_\iota\left(e\right)^{-\gamma_\iota}\mathcal{C}_{e_\iota}\left(e\right) = B_\iota e^{\varphi}.$$

Consider the LHS and recall the definition of the consumption aggregator in (1). This yields $\mathcal{C}_\iota\left(e\right) = \mathcal{C}\left(e\right)^{1/\iota}$. Taking the derivative w.r.t. e in turn yields $\mathcal{C}_{e_\iota}\left(e\right) = (1/\iota)\,\mathcal{C}\left(e\right)^{1/\iota-1}\mathcal{C}_e\left(e\right)$. Hence we obtain

$$\mathcal{C}_\iota\left(e\right)^{-\gamma_\iota}\mathcal{C}_{e_\iota}\left(e\right) = \mathcal{C}(e)^{-\frac{1}{\iota}[1-\iota(1-\gamma)]}\frac{1}{\iota}\mathcal{C}\left(e\right)^{\frac{1}{\iota}-1}\mathcal{C}_e\left(e\right) = \mathcal{C}\left(e\right)^{-\gamma}\mathcal{C}_e\left(e\right)\frac{1}{\iota},$$

and therefore the first order condition coincides after rescaling the disutility parameter.

A.1.2 IA Preferences

Proof of Proposition 2.

Proof. Differentiating the IA indirect utility function (3) yields

$$u_e\left(e;\Lambda,\overline{p}\right) = (1-\iota)\mathbf{B}(p)^{-\iota}\left(e-\mathbf{A}\left(p\right)\right)^{\iota-1},\ u_{ee}\left(e;\Lambda,\overline{p}\right) = -(\iota-1)^2\mathbf{B}(p)^{-\iota}\left(e-\mathbf{A}\left(p\right)\right)^{\iota-2}.$$

The coefficient of RRA follows as $\gamma\left(e;\Lambda,\overline{p}\right) = (\iota-1)e/(e-\mathbf{A}\left(p\right))$, and thus

$$\frac{\partial\gamma\left(e;\Lambda,\overline{p}\right)}{\partial e} = (\iota-1)\frac{\mathbf{A}\left(p\right)}{\left[e-\mathbf{A}\left(p\right)\right]^2} < 0 \quad \text{for } \mathbf{A}\left(p\right) > 0. \qquad \square$$

Proof of Corollary 2.

Proof. As shown in the proof of Corollary 1, labor supply falling requires $\gamma(e;\Lambda,\overline{p}) > 1$. As $\iota > 0$, a necessary condition for labor supply to fall is that $\mathbf{A}(p) > 0$. □

A.1.3 Cardinalization

Section 2.2 establishes that, starting from a CRRA benchmark with homothetic preferences and an identity cardinalization $V(.) = \text{id}(.)$, accounting for non-homotheticities typically implies DRRA. This section generalizes this result to other cardinalizations.

Let $W(e,n) \equiv V(u(e) - v(n))$, where the function $V(.)$ is thrice differentiable. Relative risk-aversion $\Gamma(e)$ is given by

$$\Gamma(e) \equiv -\frac{W_{ee}(e,n)e}{W_e(e,n)} = \gamma(e) - \frac{V''(u(e)-v(n))u_e(e)e}{V'(u(e)-v(n))},$$

where $\gamma(e)$ is the relative risk-aversion under the identity cardinalization. A sufficient condition for risk-aversion to be positive is that $V'(.) > 0$ and $V''(.) < 0$, which we assume next.

We want to show that for any cardinalization $V(.)$, $\Gamma_e^{nh}(e) < \Gamma_e^h(e)$, where $\Gamma^{nh}(e)$ denotes relative risk aversion with non-homothetic u, while $\Gamma^h(e)$ denotes relative risk aversion with homothetic u. That is, for any cardinalization, accounting for non-homotheticities "strengthens" the DRRA property. To do so, we compute $\Gamma_e(e)$ and obtain:

$$\begin{aligned}\Gamma_e(e) &= \gamma_e(e) + u_e\left(1-\gamma(e)\right)\frac{-V''}{V'} + u_e^2 e\left(\frac{-V''}{V'} - \frac{V'''}{-V''}\right)\left(\frac{-V''}{V'}\right)\\ &= \underbrace{\gamma_e(e)}_{\text{I}} + \underbrace{u_e\left(1-\gamma(e)\right)\left(\frac{-V''}{V'}\right)}_{\text{II}} + \underbrace{u_e^2 e\left(\frac{-V''}{V'}\right)'}_{\text{III}}\end{aligned} \tag{12}$$

Let us compare the non-homothetic and homothetic cases to establish that $\Gamma_e^{nh}(e) < \Gamma_e^h(e)$ at each level of expenditure e. Note that one can rescale the homothetic function u by a scalar such that $V(u^{nh}(e) - v(n)) = V(u^h(e) - v(n))$.

We focus on the case where the following assumptions hold: (1) $\gamma_e^{nh}(e) < 0$ while $\gamma_e^h(e) = 0$, that is, u^{nh} features DRRA; (2) labor supply falls with growth, that is, $\gamma^h(e) = \gamma > 1$ and $\gamma^{nh}(e) > 1$; (3) $\gamma^{nh}(e) > \gamma^h e$ and $u_e^{nh}(e) > u_e^h(e)$, an assumption that trivially holds for IA preferences when $\mathbf{A}(p) > 0$. Then, considering the three terms in (12), we have:

- $\text{I}^{nh} < \text{I}^h = 0$ as u^{nh} features DRRA while u^h features CRRA;
- $\text{II}^{nh} < \text{II}^h$ as $-V''/V' > 0$ and $u_e^{nh}\left(1-\gamma^{nh}(e)\right) < u_e^h\left(1-\gamma\right) < 0$;
- For term III, note that $-V''/V'$ is absolute risk aversion ARA^V; thus, if $V(.)$ features

decreasing or constant absolute risk aversion, we have $\text{III}^{nh} \leq \text{III}^{h}$ as $u_e^{nh} > u_e^h$.

Thus, $V(.)$ being CARA or DARA is a sufficient condition for $\Gamma_e^{nh} < \Gamma_e^h$. That is, if $V(.)$ does not feature too strongly increasing ARA, considering non-homotheticities translates into a relative risk aversion that is more decreasing in expenditure. Equation (12) further puts a bound on how strongly increasing ARA^V can be:

$$\left(\text{ARA}^V\right)' < \frac{\left(u_e^{nh}\left(\gamma^{nh}(e)-1\right) - u_e^h(\gamma-1)\right)\text{ARA}^V - \gamma_e^{nh}(e)}{e(u_e^{nh}-u_e^h)(u_e^{nh}+u_e^h)}$$

Note that the upper-bound defined by the right-hand side of this inequality is positive, and increasing as $\gamma^{nh}(e)$ gets larger and $\gamma_e^{nh}(e)$ gets more negative, that is, when non-homotheticities in u^{nh} get stronger.

A.2 Optimal Income Taxes

A.2.1 Household Behavior Given Taxes

We first describe households' optimal policies given taxes. We suppress dependence on p to ease notation.

Relation between u_e and u_Λ. Denote the Lagrangian multiplier associated with problem (Step 2) by μ. Applying the envelope theorem yields, omitting arguments,

$$u_e = \mu \quad \text{and} \quad u_\Lambda = \mu \sum_j \frac{p_j}{\Lambda^2} c_j = \mu \frac{e}{\Lambda}, \quad \text{and thus} \quad u_e = u_\Lambda \Lambda / e.$$

Since this relation holds for each e, we can take derivatives, which yields

$$u_{ee} = \frac{u_{\Lambda e}\Lambda e - u_\Lambda \Lambda}{e^2} = \Lambda\left(\frac{u_{\Lambda e}}{e} - \frac{u_\Lambda}{e^2}\right) \Leftrightarrow u_{e\Lambda} = \frac{e}{\Lambda}u_{ee} + u_\Lambda \frac{1}{e} = \frac{e}{\Lambda}u_{ee} + u_e \frac{1}{\Lambda}. \tag{13}$$

Labor supply decision. The first-order condition (FOC) and second-order condition (SOC) of (Step 1) read as:

$$-Bn^\varphi + u_e(e;\mathcal{T},\Lambda)(1-\mathcal{T}')\theta = 0, \tag{14}$$

$$-B\varphi n^{\varphi-1} + u_{ee}(e;\mathcal{T},\Lambda)\left((1-\mathcal{T}')\theta\right)^2 - u_e(e;\mathcal{T},\Lambda)\mathcal{T}''\theta^2 < 0. \tag{15}$$

Denoting $\rho(y) \equiv -\frac{d\log(1-\mathcal{T}'(y))}{d\log(y)} = \frac{\mathcal{T}''(y)y}{(1-\mathcal{T}'(y))}$, the SOC can be rewritten as

$$-\left(\varphi + \gamma(e;\mathcal{T},\Lambda)\frac{(1-\mathcal{T}')n\theta}{e} + \rho(n\theta)\right) < 0.$$

Wealth effect on labor supply. First, we derive the response of labor supply to an increase in the intercept of the tax $\mathcal{T}(0)$. Implicit differentiation yields:

$$\frac{\partial n}{\partial \mathcal{T}(0)} = -\frac{-u_{ee}(e;\mathcal{T},\Lambda)(1-\mathcal{T}')\theta}{-\varphi B n^{\varphi-1} + u_{ee}(e;\mathcal{T},\Lambda)(1-\mathcal{T}')^2\theta^2 - u_e(e;\mathcal{T},\Lambda)\mathcal{T}''\theta^2}.$$

Using equation (15) and the definition of $\gamma(e;\Lambda)$ and rearranging, we obtain:

$$\frac{\partial n}{\partial \mathcal{T}(0)} = \frac{\gamma(e;\Lambda)\frac{n}{e}}{\varphi + \gamma(e;\Lambda)\frac{n}{e}(1-\mathcal{T}')\theta + \rho(y)}.$$

Equation (5) then immediately follows from its definition.

A.2.2 Optimal Income Tax Formula

We first derive elasticities with respect to type and to $1-\mathcal{T}'$.

Elasticity w.r.t. type. Implicit differentiation of the individual FOC yields:

$$\frac{\partial n}{\partial \theta} = -\frac{u_{ee}(e;\mathcal{T},\Lambda)(1-\mathcal{T}')^2\theta n + u_e(e;\mathcal{T},\Lambda)(1-\mathcal{T}') - u_e(e;\mathcal{T},\Lambda)\mathcal{T}''\theta n}{-\varphi B n^{\varphi-1} + u_{ee}(e;\mathcal{T},\Lambda)(1-\mathcal{T}')^2\theta^2 - u_e(e;\mathcal{T},\Lambda)\mathcal{T}''\theta^2}.$$

Rearranging and using again equation (14), we obtain:

$$\varepsilon_{n,\theta} = \frac{dn}{d\theta}\frac{\theta}{n} = \frac{1}{\varphi}\frac{\left(1-\gamma(e;\mathcal{T},\Lambda)\frac{(1-\mathcal{T}')\theta n}{e} - \rho(y)\right)}{1+\frac{\gamma(e;\mathcal{T},\Lambda)}{\varphi}\frac{(1-\mathcal{T}')\theta n}{e} + \frac{\rho(y)}{\varphi}}. \tag{16}$$

Elasticity w.r.t. to $1-\mathcal{T}'$. The derivation is analogous to the derivation of $\varepsilon_{n,\theta}$ and one obtains:

$$\varepsilon_{n,1-\mathcal{T}'} = \frac{1}{\varphi}\times\frac{1}{1+\frac{\gamma(e;\Lambda)}{\varphi}\frac{(1-\mathcal{T}')\theta n}{e} + \frac{\rho(y)}{\varphi}}. \tag{17}$$

Proof of Lemma 2.

Proof. The Lagrangian of the government's problem is:

$$\begin{aligned}\mathcal{L} = &\int_{\underline{\theta}}^{\overline{\theta}} u\left[n(\theta;\mathcal{T}(\cdot,\Lambda),\Lambda,p)\theta - \mathcal{T}\left(n(\theta;\mathcal{T}(\cdot,\Lambda),\Lambda,p)\theta\right);\Lambda,p\right] w(\theta)f(\theta)d\theta \\ &- B\int_{\underline{\theta}}^{\overline{\theta}} \frac{n(\theta;\mathcal{T}(\cdot,\Lambda),\Lambda,p)^{1+\varphi}}{1+\varphi}w(\theta)f(\theta)d\theta + \lambda\int_{\underline{\theta}}^{\overline{\theta}} \mathcal{T}(n(\theta;\mathcal{T}(\cdot,\Lambda),\Lambda,p)\theta)f(\theta)d\theta - \lambda G,\end{aligned}$$

We follow the heuristic approach going back to Saez (2001) for deriving the optimality condition for the marginal tax rate. Consider an increase in the marginal tax rate by $d\mathcal{T}'$ within a small interval $[(y(\theta^*;\mathcal{T}(\cdot,\Lambda),\Lambda), y(\theta^*;\mathcal{T}(\cdot,\Lambda),\Lambda)+dy]$. The mass of people affected by this increase in the marginal tax rate is approximately given by $h(y(\theta^*;\mathcal{T},\Lambda);\mathcal{T},\Lambda)\times dy$ where h is the density function of the endogenous income distribution defined through $F(\theta^*) = H(y(\theta^*;\mathcal{T},\Lambda))$ and hence $h(y(\theta^*;\mathcal{T},\Lambda))y_\theta(\theta^*;\mathcal{T},\Lambda) = f(\theta^*)$. We therefore have

$$h(y(\theta^*;\mathcal{T},\Lambda);\mathcal{T},\Lambda)\times dy = \frac{f(\theta^*)dy}{y_\theta(\theta^*;\mathcal{T},\Lambda)}.$$

Note that each individual affected by the increase in the marginal tax rate changes their earnings by

$$\frac{\partial y(\theta^*;\mathcal{T},\Lambda)}{\partial \mathcal{T}'}d\mathcal{T}' = -\varepsilon_{y,1-\mathcal{T}'}(\theta^*;\mathcal{T},\Lambda)\frac{y(\theta^*;\mathcal{T},\Lambda)}{1-\mathcal{T}'(y(\theta^*;\mathcal{T},\Lambda);\Lambda)}d\mathcal{T}'.$$

The "substitution effect", that is, the welfare effect of this labor supply change, is given by

$$\begin{aligned} dS(\theta^*;\mathcal{T},\Lambda) &= -\lambda\frac{\mathcal{T}'(y(\theta^*;\mathcal{T},\Lambda);\Lambda)}{1-\mathcal{T}'(y(\theta^*;\mathcal{T},\Lambda);\Lambda)}\frac{\varepsilon_{y,1-\mathcal{T}'}(\theta^*;\mathcal{T},\Lambda)}{\varepsilon_{y,\theta}(\theta^*;\mathcal{T},\Lambda)}\theta^* d\mathcal{T}' f(\theta^*)dy \\ &= -\lambda\frac{\mathcal{T}'(y(\theta^*;\mathcal{T},\Lambda);\Lambda)}{1-\mathcal{T}'(y(\theta^*;\mathcal{T},\Lambda);\Lambda)}\frac{1}{\varphi+1}\theta^* d\mathcal{T}' f(\theta^*)dy, \end{aligned}$$

where the last equality uses the expressions for the elasticities in (16) and (17) and $\varepsilon_{y,\theta} = 1+\varepsilon_{n,\theta}$ as well as $\varepsilon_{y,1-\mathcal{T}'} = \varepsilon_{n,1-\mathcal{T}'}$.

Note that one can express the tax formula in terms of the distribution of income instead of types. When doing so, the compensated labor supply elasticity $\varepsilon_{y,1-\mathcal{T}'}$ appears explicitly in the formula, and does change with Λ. However, the density of income also changes with Λ in the same way as the elasticity, so that the two effects cancel out. This is why only the constant Frisch elasticity φ^{-1} appears in the formula with types that we use in Lemma (2).

Next, there is a mechanical effect: households with $\theta > \theta^*$ pay $d\mathcal{T}'dy$ more taxes:

$$dM(\theta^*;\mathcal{T},\Lambda) = d\mathcal{T}'dy \times \int_{\theta^*}^{\overline{\theta}} \left(\lambda - u_e(\theta;\mathcal{T},\Lambda)w(\theta)\right) f(\theta)d\theta,$$

where $u_e(\theta;\Lambda) = u_e(e(\theta;\Lambda);\Lambda)$. Finally, there is an income effect: all households with $\theta > \theta^*$ now get poorer by $d\mathcal{T}'dy$ and change their income, which has a tax revenue effect:

$$dI(\theta^*;\mathcal{T},\Lambda) = d\mathcal{T}'dy \times \lambda\int_{\theta^*}^{\overline{\theta}} \mathcal{T}'(y(\theta;\mathcal{T},\Lambda);\Lambda)\eta(\theta;\mathcal{T},\Lambda)f(\theta)d\theta.$$

If the tax schedule is optimal, all welfare effects have to add up to zero: $dS(\theta^*;\mathcal{T},\Lambda)) + dM(\theta^*;\mathcal{T},\Lambda) + dI(\theta^*;\mathcal{T},\Lambda) = 0$. This then yields the optimality condition as in Lemma 2. □

A.2.3 Homothetic Benchmark

A direct proof can be derived in two steps: first, show that incomes and expenditures grow with Λ at rate $\alpha \equiv (1-\gamma)/(\varphi+\gamma)$ equal for all θ when marginal wedges, that is, marginal tax rates at productivity θ, remain constant as Λ grows; second, show that when incomes and expenditures grow at constant rate α, both sides of the optimality condition stated in Lemma 2 are unaffected by Λ when marginal wedges are constant.

An alternative proof defines the income-tax reform that implements constant marginal wedges as Λ grows and shows its optimality. We follow this approach as the definition of the tax reform itself may be useful to the reader.

Proof of Proposition 3 and Corollary 3. *Tax Reform.*—Consider a marginal increase in Λ by $d\Lambda$, and let $g \equiv d\Lambda/\Lambda$. We denote a tax reform that accompanies this increase in Λ by

$$\forall y: \; d\mathcal{T}(y;\Lambda) \equiv \lim_{g\to 0} \frac{1}{g} \left\{ \mathcal{T}(y;\Lambda(1+g)) - \mathcal{T}(y;\Lambda) \right\}.$$

For any variable $v(\theta;\mathcal{T},\Lambda)$, we denote its relative change due to growth g and the accompanying tax reform $d\mathcal{T}$ as

$$\hat{v}(\theta;\mathcal{T},\Lambda,d\mathcal{T}) \equiv \lim_{g\to 0} \frac{1}{g} \frac{v\big(\theta;\mathcal{T}(\cdot;\Lambda) + g \times d\mathcal{T}(\cdot;\Lambda), \Lambda(1+g)\big)}{v\big(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda\big)} - 1. \tag{18}$$

Proposition: Tax Reform Formulation.—Assume preferences $u(e;\Lambda)$ satisfy CRRA in Preferences 1 and 2. The optimal tax reform, which we denote $d\tilde{\mathcal{T}}$, to a marginal change in Λ is characterized by:

$$\forall y: \; d\tilde{\mathcal{T}}(y;\Lambda) = \left(\mathcal{T}(y;\Lambda) - \mathcal{T}'(y;\Lambda)y\right)\alpha, \tag{19}$$

where $\alpha \equiv (1-\gamma)/(\varphi+\gamma)$. The resulting allocation is such that:

1. Expenditures and incomes grow at rate α $\forall\theta: \hat{y}(\theta;\mathcal{T},\Lambda,d\tilde{\mathcal{T}}) = \hat{e}(\theta;\mathcal{T},\Lambda,d\tilde{\mathcal{T}}) = \alpha$.

2. Optimal marginal and average tax rates are constant $\forall\theta$:

$$\forall\theta: \; \mathcal{T}'(y(\theta;\Lambda(1+g));\Lambda(1+g)) = \mathcal{T}'(y(\theta;\Lambda);\Lambda),$$
$$\forall\theta: \; \frac{\mathcal{T}(y(\theta;\Lambda(1+g));\Lambda(1+g))}{y(\theta;\Lambda(1+g))} = \frac{\mathcal{T}(y(\theta;\Lambda),\Lambda)}{y(\theta;\Lambda)}.$$

Proof. We proceed in 4 steps: (1) we show that incomes grow at rate $\alpha \equiv (1-\gamma)/(\varphi+\gamma)$ in response to growth $g \to 0$ and its accompanied tax reform (19); (2) we show that expenditures also grow at rate α; (3) we show that marginal and average tax rates stay constant given θ;

and (4) we show that, given steps (1-3), tax reform (19) is indeed optimal.

Step 1. First, note that tax reform (19) implies a marginal change in the absolute level of the tax payment for income level y by

$$d\tilde{\mathcal{T}}(y;\Lambda) = \frac{1-\gamma}{\varphi+\gamma}\left(\mathcal{T}(y;\Lambda) - \mathcal{T}'(y;\Lambda)y\right). \tag{20}$$

The implied change in the marginal tax rate is then given by

$$d\tilde{\mathcal{T}}'(y;\Lambda) = \frac{1-\gamma}{\varphi+\gamma}\left(\mathcal{T}'(y;\Lambda) - \mathcal{T}'(y;\Lambda) - \mathcal{T}''(y;\Lambda)y\right) = -\frac{1-\gamma}{\varphi+\gamma}\mathcal{T}''(y;\Lambda)y. \tag{21}$$

Now consider a small perturbation of the individual first-order conditions by $g = \frac{d\Lambda}{\Lambda}$ and associated change in the absolute tax level as defined in (20) and in the marginal tax rate as defined in (21). The adjustment of labor supply dn such that the FOC still holds is then defined by:

$$\begin{aligned} &SOC dn + u_{e\Lambda}(e;\mathcal{T},\Lambda)(1-\mathcal{T}')\theta\Lambda g - u_e(e;\mathcal{T},\Lambda)\theta\left(-\frac{1-\gamma}{\varphi+\gamma}\mathcal{T}''y\right)g \\ &- u_{ee}(e;\mathcal{T},\Lambda)(1-\mathcal{T}')\theta\frac{1-\gamma}{\varphi+\gamma}\left(\mathcal{T}-\mathcal{T}'y\right)g = 0, \end{aligned}$$

with SOC defined in (15). Solving for dn and using (13) yields (omitting arguments)

$$\frac{dn}{g} = \frac{-\left(u_{ee}e+u_e\right)(1-\mathcal{T}')\theta - u_e\theta\frac{1-\gamma}{\varphi+\gamma}\mathcal{T}''y + u_{ee}(1-\mathcal{T}')\theta\frac{1-\gamma}{\varphi+\gamma}\left(\mathcal{T}-\mathcal{T}'y\right)}{-B\varphi n^{\varphi-1} + u_{ee}\left((1-\mathcal{T}')\theta\right)^2 - u_e\mathcal{T}''\theta^2}.$$

Collecting u_{ee} and u_e terms and invoking the FOC $u_e\theta(1-\mathcal{T}') = Bn^{\varphi}$ yields:

$$\hat{n}(\theta;\mathcal{T},\Lambda,d\mathcal{T}) = \lim_{g\to 0}\frac{\frac{dn}{g}}{n} = \frac{\gamma(\theta;\Lambda)\left(1-\frac{1-\gamma}{\varphi+\gamma}\left(\mathcal{T}-\mathcal{T}'y\right)\frac{1}{e}\right) - \left(1+\frac{1-\gamma}{\varphi+\gamma}\frac{\mathcal{T}''}{1-\mathcal{T}'}y\right)}{-\varphi-\gamma(\theta;\Lambda)\frac{(1-\mathcal{T}')y}{e} - \frac{\mathcal{T}''}{1-\mathcal{T}'}y}.$$

Rearranging and using $y = e + \mathcal{T}$ and hence $1 + \frac{\mathcal{T}}{e} = \frac{y}{e}$ implies

$$\hat{n}(\theta;\mathcal{T},\Lambda,d\mathcal{T}) = \frac{1-\gamma}{\varphi+\gamma}\left(1+\frac{\frac{\varphi+\gamma}{1-\gamma}\left(1-\gamma(\theta;\Lambda)\right)-\gamma(\theta;\Lambda)-\varphi}{\varphi+\gamma(\theta;\Lambda)\frac{(1-\mathcal{T}')y}{e}+\frac{\mathcal{T}''}{1-\mathcal{T}'}y}\right). \tag{22}$$

In the homothetic case where $\gamma(\theta;\Lambda) = \gamma\ \forall(\theta,\Lambda)$, this implies $dy/y = \alpha$.

Step 2. As $e = y - \mathcal{T}(y;\Lambda)$, the change in expenditure is given by

$$\frac{de}{e} = \frac{dy\left(1-\mathcal{T}'(y;\Lambda)\right) - d\tilde{\mathcal{T}}(y;\Lambda)}{e} = \frac{\alpha y(1-\mathcal{T}'(y;\Lambda)) - \alpha y(\mathcal{T}(y;\Lambda)-\mathcal{T}'(y;\Lambda))}{e} = \alpha.$$

Step 3. For a given θ, the average tax rate is constant, as $\mathcal{T}(y;\Lambda)/y = (y-e)/y$ and we have shown that both y and e grow at the same rate α.

Next, we turn to the marginal tax rate. The new tax schedule is defined as

$$\lim_{g\to 0}\mathcal{T}(y;\Lambda(1+g)) = \mathcal{T}(y;\Lambda) + \lim_{g\to 0} g\alpha\left(\mathcal{T}(y;\Lambda) - \mathcal{T}'(y;\Lambda)y\right),$$

$$\text{and hence,}\quad \lim_{g\to 0}\mathcal{T}'(y;\Lambda(1+g)) = \mathcal{T}'(y;\Lambda) - \lim_{g\to 0} g\alpha\mathcal{T}''(y;\Lambda)y. \tag{23}$$

$$\text{We want to show that}\quad \lim_{g\to 0}\mathcal{T}'\left(y(1+\alpha g);\Lambda(1+g)\right) = \mathcal{T}'(y;\Lambda). \tag{24}$$

Evaluating (23) at $y(1+\alpha g)$, we obtain

$$\begin{aligned}
\lim_{g\to 0}\mathcal{T}'(y(1+\alpha g);\Lambda(1+g)) &= \lim_{g\to 0}\left[\mathcal{T}'(y(1+\alpha g);\Lambda) - g\alpha\mathcal{T}''(y(1+\alpha g);\Lambda)y(1+\alpha g)\right] \\
&= \mathcal{T}'(y;\Lambda) + \lim_{g\to 0}\left[g\alpha y\mathcal{T}''(y(1+\alpha g);\Lambda) - g\alpha\mathcal{T}''(y(1+\alpha g);\Lambda)y(1+\alpha g)\right] \\
&= \mathcal{T}'(y;\Lambda) - \lim_{g\to 0} g^2\alpha^2\mathcal{T}''(y(1+\alpha g);\Lambda)y = \mathcal{T}'(y;\Lambda).
\end{aligned}$$

Step 4. We show that tax reform (19) satisfies the government's optimality conditions at the allocation it implements.

We start with the distributional gains term. As $e(\theta;\Lambda)$ grows at rate α $\forall\theta$,

$$\begin{aligned}
\hat{D}(\theta^*;\mathcal{T},\Lambda,d\tilde{\mathcal{T}}) =& \frac{\int_{\underline{\theta}}^{\bar{\theta}}\left(u_{ee}(x;\Lambda)\alpha e(x;\Lambda) + u_{e\Lambda}(x;\Lambda)\Lambda\right)w(x)dF(x)}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(x;\Lambda)w(x)dF(x)} \\
&- \frac{\int_{\theta^*}^{\bar{\theta}}\left(u_{ee}(x;\Lambda)\alpha e(x;\Lambda) + u_{e\Lambda}(x;\Lambda)\Lambda\right)w(x)dF(x)}{\int_{\theta^*}^{\bar{\theta}} u_e(x;\Lambda)w(x)dF(x)}.
\end{aligned}$$

where the "hat-notation" is as defined in (18). Using (13) and rearranging,

$$\begin{aligned}
\hat{D}(\theta^*;\mathcal{T},\Lambda,d\tilde{\mathcal{T}}) =& \frac{\int_{\underline{\theta}}^{\bar{\theta}}\left(-\gamma(\theta;\Lambda)u_e(x;\Lambda)(1+\alpha) + u_e(x;\Lambda)\right)w(x)dF(x)}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(x;\Lambda)w(x)dF(x)} \\
&- \frac{\int_{\theta^*}^{\bar{\theta}}\left(-\gamma(\theta;\Lambda)u_e(x;\Lambda)(1+\alpha) + u_e(x;\Lambda)\right)w(x)dF(x)}{\int_{\theta^*}^{\bar{\theta}} u_e(x;\Lambda)w(x)dF(x)}.
\end{aligned} \tag{25}$$

For the homothetic case, $\gamma(\theta;\Lambda) = \gamma$ for all θ yields $\hat{D}(\theta^*;\mathcal{T},\Lambda,d\tilde{\mathcal{T}}) = 0$.

We turn to the efficiency costs term. We need to show that $\eta(\theta;\mathcal{T},\Lambda) = \eta(\theta;\mathcal{T} + g\times d\tilde{\mathcal{T}},\Lambda(1+g))$. As $e(\theta;\Lambda)$ and $y(\theta;\Lambda)$ grow at rate α, it requires that $\rho(y;\Lambda) = \rho(y(1+\alpha g);\Lambda(1+g))$. Make use of the fact that (24) has to hold for each value of y implying $\mathcal{T}''(y(1+\alpha g);\Lambda(1+g))(1+\alpha g) = \mathcal{T}''(y;\Lambda)$, which immediately implies that

$\rho(y\,(1+\alpha g)\,;\Lambda\,(1+g))=\rho(y;\Lambda).$ □

A.2.4 NH Preferences

Proof of Proposition 4.

Proof. Derivating $D(.)$ with respect to Λ yields

$$D_\Lambda(\theta^*;\mathcal{T},\Lambda)=-\frac{\int_{\theta^*}^{\bar\theta}\frac{\partial u_e(\theta;\Lambda)}{\partial\Lambda}w(\theta)\frac{dF(\theta)}{1-F(\theta^*)}\int_{\underline\theta}^{\bar\theta}u_e(\theta;\Lambda)w(\theta)dF(\theta)-\int_{\underline\theta}^{\bar\theta}\frac{\partial u_e(\theta;\Lambda)}{\partial\Lambda}w(\theta)dF(\theta)\int_{\theta^*}^{\bar\theta}u_e(\theta;\Lambda)w(\theta)\frac{dF(\theta)}{1-F(\theta^*)}}{\left(\int_{\underline\theta}^{\bar\theta}u_e(\theta;\Lambda)w(\theta)dF(\theta)\right)^2}$$

We define $\mathbb{E}_{\tilde w}$ the expectation over the distribution $\tilde w$ defined as: $\tilde w(\theta^*)\equiv w(\theta)u_e(\theta^*;\Lambda)/\int_{\underline\theta}^{\bar\theta}u_e(\theta;\Lambda)w(\theta)dF(\theta)$, which integral sums up to 1. Rearranging D_Λ we obtain:

$$\text{sign}(D_\Lambda(\theta^*;\mathcal{T},\Lambda))=\frac{1}{\Lambda}\left(-\mathbb{E}_{\tilde w}[\gamma(\theta,\Lambda)]+\mathbb{E}_{\tilde w}[\gamma(\theta,\Lambda)|\theta\geq\theta^*]\right)$$

which is negative as DRRA implies that $\gamma(\theta,\Lambda)$ is decreasing in θ. □

Proof of Lemma 3.

Proof. Fixing income and expenditure distributions as well as taxes, rewriting the equation (5), which defines income effects, yields:

$$\eta(\theta;\Lambda)=\frac{\frac{y}{e}}{\frac{1}{\gamma(e;\Lambda)}\left(\varphi+\frac{\mathcal{T}''(y)y}{1-\mathcal{T}'(y)}\right)+\frac{y}{e}(1-\mathcal{T}'(y))}$$

Thus, $\eta(\theta,\Lambda)$ only depends on Λ through $\gamma(e;\Lambda)$: under Assumption 4, $\eta(\theta;\Lambda)$ is an increasing function of $\gamma(\theta;\Lambda)$. It is easy to show that $\gamma_\Lambda(e;\Lambda)=\frac{e}{\Lambda}\gamma_e(e;\Lambda)<0$, which implies that $\eta_\Lambda(\theta;\Lambda)<0$. □

Proof of Proposition 5.

Proof. Taking the derivative of $E(\theta^*;\mathcal{T},\Lambda)$, fixing income and expenditure distributions and taxes, we obtain that:

$$\text{sgn}(E_\Lambda)=-\left[\int_{\theta^*}^{\bar\theta}\mathcal{T}'(y(\theta;\Lambda);\Lambda)\eta_\Lambda(\theta;\Lambda)\frac{dF(\theta)}{1-F(\theta^*)}\right]+\left[\int_{\underline\theta}^{\bar\theta}\mathcal{T}'(y(\theta;\Lambda);\Lambda)\eta_\Lambda(\theta;\Lambda)dF(\theta)\right](1-E),$$

where the first term captures the efficiency cost of taxes while the second capture the efficiency cost of transfers. The proposition follows from $\eta_\Lambda(\theta;\Lambda)<0$. □

Proof of Proposition 6.

Proof. Differentiating the first-order condition of the household problem gives:

$$\frac{n_\Lambda}{n} = \frac{1}{\Lambda} \frac{1-\gamma(\theta,\Lambda)}{\varphi + \gamma(\theta,\Lambda)\frac{\theta n}{\theta n - \mathcal{T}}(1-\mathcal{T}') + \theta n \frac{\mathcal{T}''}{1-\mathcal{T}'}} \tag{26}$$

Note that $n_\Lambda/n > 0$ as long as $\gamma(\theta,\Lambda) > 1$—as long as the SOC of the households' problem is negative: labor supply falls with growth when relative risk-aversion is larger than one.

Evaluating equation (26) under the Laissez-Faire and differentiating w.r.t. θ yields:

$$\frac{\partial}{\partial\theta}\left(\frac{n_\Lambda}{n}\right) = -\frac{\gamma_\theta(\theta,\Lambda)}{\Lambda}\frac{\varphi+1}{(\varphi+\gamma(\theta,\Lambda))^2} > 0,$$

as $\text{sgn}(\gamma_\theta(.)) = \text{sgn}(\gamma_e(.)) < 0$ under DRRA. Similarly, evaluating equation (26) at a loglinear function and differentiating with respect to θ, we obtain:

$$\text{sgn}\left(\frac{\partial}{\partial\theta}\left(\frac{n_\Lambda}{n}\right)\right) = -\gamma_\theta(\theta,\Lambda)\left(\varphi+\tau+1\right),$$

which is unambiguously positive when $\tau + \varphi \geq -1$, a condition that holds when the SOC of the household's problem holds. □

A.3 Laissez-Faire

Proof of Proposition 7.

Proof. We start from a distribution of Pareto weights such that $\mathcal{T}'(y(\theta,\Lambda),\Lambda) = \mathcal{T}(y(\theta,\Lambda),\Lambda) = 0$ for all θ—that is, the Laissez-Faire allocation is optimal at Λ. We consider a marginal increase in Λ by $d\Lambda$ and define $g \equiv d\Lambda/\Lambda$. To ease notation, we define two objects. First, let $d\tau(\theta)$ denote the change in the optimal marginal tax rate of type θ due to g:

$$d\tau(\theta) \equiv \mathcal{T}'(y(\theta,\Lambda(1+g)),\Lambda(1+g)).$$

Note that $d\tau(\theta)$ also captures the level of the optimal marginal rate after the rise in Λ as its starting point was zero (Laissez-Faire allocation). Second, let $d\mathrm{T}(\theta)$ denote the change in the optimal tax payment of type θ, as well as its level after the rise in Λ:

$$d\mathrm{T}(\theta) \equiv \mathcal{T}(y(\theta,\Lambda(1+g)),\Lambda(1+g)).$$

Further, we suppress dependence on Λ of all allocation variables. We prove Proposition 7 in two steps. First, we show that either marginal tax rates are negative or, if they are positive, average tax rates are falling in income. Second, we show that the lump-sum component of the tax schedule turns positive after the rise in Λ.

Part 1: Negative marginal tax rates or falling average tax rates. The optimal tax schedule after the rise in Λ still satisfies the optimality condition stated in Lemma 2. Let $X \equiv 1 + \int_{\underline{\theta}}^{\bar{\theta}} d\tau(\theta)\eta(\theta)dF(\theta)$ and $Y \equiv \int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta)w(\theta)dF(\theta)$.[26] The optimality condition in Lemma 2 can be written as

$$\frac{d\tau(\theta^*)}{1-d\tau(\theta^*)}\frac{\theta^* f(\theta^*)}{1+\varphi} = 1 - F(\theta^*) - \int_{\theta^*}^{\bar{\theta}} \left(\frac{X}{Y}u_e(\theta)w(\theta) - d\tau(\theta)\eta(\theta)\right) dF(\theta).$$

Derivativing both sides and rearranging yields

$$\frac{\partial\left(\frac{d\tau(\theta^*)}{1-d\tau(\theta^*)}\right)}{\partial\theta}\frac{\theta^*}{1+\varphi} + \frac{d\tau(\theta^*)}{1-d\tau(\theta^*)}\frac{1+\theta^*\frac{f'(\theta^*)}{f(\theta^*)}}{1+\varphi} = -1 + \frac{X}{Y}\omega(\theta^*)u_e(\theta^*) - d\tau(\theta^*)\eta(\theta^*). \quad (27)$$

We establish conditions under which an interval $[y(\theta_1), y(\theta_2)]$ can exist such that $d\tau(\theta_1) = d\tau(\theta_2) = 0$ and $d\tau(\theta) > 0 \; \forall\theta \in]\theta_1, \theta_2[$. Equation (27) implies that this can be optimal only if

$$\omega(\theta_1)u_e(\theta_1) > \omega(\theta_2)u_e(\theta_2), \quad (28)$$

since the LHS of (27) is positive at θ_1 and negative at θ_2. Prior to the rise in Λ, the Laissez-faire allocation is assumed to be optimal and hence, $\omega(\theta_1)u_e(\theta_2) = \omega(\theta_2)u_e(\theta_2)$. Denote by $du_e(\theta_1)$ the change in marginal utility of type θ due to the rise in Λ and its associated optimal change of the $t\&T$ system. For equation (28) to hold, it must be that

$$\frac{du_e(\theta_1)}{u_e(\theta_1)} > \frac{du_e(\theta_2)}{u_e(\theta_2)}. \quad (29)$$

We now derive explicit expressions for the change in marginal utility:

$$du_e(\theta) = u_{e\Lambda}(\theta)d\Lambda + u_{ee}(\theta)\frac{\partial y(\theta)}{\partial\Lambda}d\Lambda - u_{ee}(\theta)d\mathrm{T}(\theta) + u_{ee}(\theta)\left(\frac{\partial y}{\partial\mathcal{T}'}d\tau(\theta) + \frac{\partial y(\theta)}{\partial\mathcal{T}}d\mathrm{T}(\theta)\right).$$

It consists of four terms: (1) the direct effect of the rise in Λ on marginal utility; (2) the effect due to the labor supply change; (3) the direct effect of the change in the $t\&T$ system; and (4) the effect due to the labor supply change in response to the change in the $t\&T$ system. Using (13), $\varepsilon_{y,\Lambda} = \varepsilon_{n,\theta}$ and the fact that in the Laissez-Faire we have $e(\theta) = y(\theta)$, we can rewrite the first two terms as $u_{ee}\frac{ed\Lambda}{\Lambda} + \frac{u_e d\Lambda}{\Lambda} + u_{ee}\frac{1-\gamma(\theta)}{\varphi+\gamma(\theta)}\frac{d\Lambda}{\Lambda}y$. Dividing by u_e, we can

[26] Note that X represents the planner's first-order condition with respect to the lump-sum component of the tax schedule. A value $X < 0$ would imply that an increase in the lump-sum transfer is self-financing. Therefore, at the optimum, we must have $X > 0$, as otherwise, it would be possible to implement a Pareto-improving increase in the lump-sum transfer.

then rewrite the whole expression as

$$\frac{du_e(\theta)}{u_e(\theta)} = g\varphi\left(\frac{1-\gamma(\theta)}{\varphi+\gamma(\theta)}\right) + \gamma(\theta)\frac{d\mathrm{T}(\theta)}{y(\theta)} + \gamma(\theta)\varepsilon_{y,1-\tau}d\tau(\theta) - \gamma(\theta)\eta(\theta)\frac{d\mathrm{T}(\theta)}{y(\theta)},$$

where we use that the baseline marginal tax rate is zero in the Laissez-Faire: $\mathcal{T}'(y(\theta)) = 0$. In the Laissez-Faire we have $\eta(\theta) = \frac{\gamma(\theta)}{\varphi+\gamma(\theta)}$ and $\varepsilon_{y,1-\tau} = \frac{1}{\varphi+\gamma(\theta)}$, so that:

$$\frac{du_e(\theta)}{u_e(\theta)} = g\varphi\left(\frac{1-\gamma(\theta)}{\varphi+\gamma(\theta)}\right) + \gamma(\theta)\left(\frac{d\mathrm{T}(\theta)}{y(\theta)}\frac{\varphi}{\varphi+\gamma(\theta)} + \frac{d\tau(\theta)}{\varphi+\gamma(\theta)}\right). \quad (30)$$

Consider two income levels $y(\theta_2) > y(\theta_1)$ such that $d\tau(\theta_1) = d\tau(\theta_2) = 0$ and $d\tau(\theta) > 0$ $\forall\theta \in]\theta_1, \theta_2[$. For $\theta \in \{\theta_1, \theta_2\}$, equation (30) simplifies to

$$\frac{du_e(\theta)}{u_e(\theta)} = = g\varphi\frac{1-\gamma(\theta)\left(1-\frac{dT(y(\theta)}{y(\theta)g}\right)}{\varphi+\gamma(\theta)}. \quad (31)$$

Let $d\mathcal{T}_{avg}(\theta) \equiv d\mathrm{T}(\theta)/y(\theta)$. Equation (29) then implies that:

$$\frac{1-\gamma(\theta_1)\left(1-\frac{d\mathcal{T}_{avg}(\theta_1)}{g}\right)}{\varphi+\gamma(\theta_1)} > \frac{1-\gamma(\theta_2)\left(1-\frac{d\mathcal{T}_{avg}(\theta_2)}{g}\right)}{\varphi+\gamma(\theta_2)},$$

which can be rewritten as:

$$\frac{(\varphi\gamma(\theta_1)+\gamma(\theta_1)\gamma(\theta_2))\, d\mathcal{T}_{avg}(\theta_1)) - (\varphi\gamma(\theta_2)+\gamma(\theta_1)\gamma(\theta_2))\, d\mathcal{T}_{avg}(\theta_2))}{(1+\varphi)\,(\gamma(\theta_1)-\gamma(\theta_2))} > g > 0. \quad (32)$$

The goal is therefore to check if equation (32) can hold—and if so, under which conditions.

1. Consider the case with $d\mathcal{T}_{avg}(\theta_2) < 0$ and $d\mathcal{T}_{avg}(\theta_1) < 0$. Positive marginal tax rates in the interval $]\theta_1, \theta_2[$ imply $d\mathcal{T}_{avg}(\theta_2) > d\mathcal{T}_{avg}(\theta_1)$; hence the RHS of (32) is negative, which cannot happen as $g > 0$. Thus, when average tax rates are negative, marginal tax rates cannot be positive.

2. Considering the case with $d\mathcal{T}_{avg}(\theta_2) > 0$ and $d\mathcal{T}_{avg}(\theta_1) < 0$ yields the same conclusion as above.

3. Consider the case with $d\mathcal{T}_{avg}(\theta_2) > 0$ and $d\mathcal{T}_{avg}(\theta_1) > 0$. There, $\mathcal{T}_{avg}(\theta_2) - \mathcal{T}_{avg}(\theta_1)$ can be positive or negative. With marginal tax rates in the interval $]\theta_1, \theta_2[$ lower than $d\mathcal{T}_{avg}(\theta_1) > 0$, average tax rates may be declining even when marginal tax rates are positive. Thus, the RHS of (32) can be positive: equation (32) may hold. Hence, an interval with positive marginal tax rates *can* exist, but only when average tax rates are falling (sufficiently quickly) in that interval.

4. The last case, $d\mathcal{T}_{avg}(\theta_2) < 0$ and $d\mathcal{T}_{avg}(\theta_1) > 0$, cannot happen as $d\tau(\theta) > 0 \, \forall \theta \in]\theta_1, \theta_2[$.

Finally, note that Pareto efficiency precludes nonzero marginal rates at $\underline{\theta}$ and $\overline{\theta}$, so that an interval with positive marginal rates must have zero marginal rates at its bounds.

Part 2: Positive lump-sum tax element $T(0) > 0$. We prove this part by contradiction. Assume that the lump-sum component of the tax schedule is negative, i.e. $T(0) < 0$. Then, average tax rates are negative for the lowest income levels, which implies negative marginal tax rates due to Part 1 of the proof. Thus, average tax rates remain negative, implying negative marginal tax rates everywhere, which cannot be budget feasible. □

B Data

In this section, we describe the dataset we use to compute income and wealth distributions in 1950 and 2010.

B.1 SCF+

The SCF+ provides long-run data on income and wealth inequality in the United States. It is compiled by Kuhn et al. (2020), based on historical waves of the SCF. The covered time period is from 1949 to 2016.

As income components in the data, we use wages and salaries, income from professional practice and self-employment, and business and farm income. We exclude rental income, interest, dividends, and transfers, as we model asset income and transfers separately from the labor income process.

For wealth, we compute net worth as the sum of all assets minus the sum of all debts. Assets include liquid assets (checking, savings, call/money market accounts, certificates of deposit), housing and other real estate, bonds, stocks and business equity, mutual funds, cash value of life insurance, defined-contribution retirement plans, and cars. Debt consists of housing debt (debt on owner-occupied homes, home equity loans and lines of credit) and other debt (car loans, education loans, consumer loans).

We restrict the sample to the working age population, i.e. household heads aged 25 to 60. We impose that minimum household income is \$5,000 in 2010 (in 2016 dollars). In 1950, we choose the cutoff such that the ratio of minimum income to median income is the same as in 2010, which results in a cutoff of \$2,700 (in 2016 dollars).

Table C.1: Untargeted Data and Model Moments

Moment	Source	Data	Model
Agg. agriculture share 1950	Herrendorf et al. (2013)	21.5%	16.7%
Agg. goods share 1950	Herrendorf et al. (2013)	39.2%	49.0%
Agg. services share 1950	Herrendorf et al. (2013)	39.2%	34.3%
Wealth-to-income ratio 1950	Piketty et al. (2014)	3.65	3.22
Agg. fall in labor supply	Ramey et al. (2009)	5-7%	7.7%

Notes: Table C.1 summarizes a subset of untargeted data moments and their model counterparts.

C Quantitative Models

C.1 Taxes, Transfers, and Spending

Tax-and-transfer function. The $t\&T$ function used in Section 4.2.4 has been introduced in Ferriere et al. (2023). As compared to the widely used loglinear tax function, popularized by Feldstein (1969) and Heathcote et al. (2017), it allows to better jointly match the bottom and the top of the tax distribution. Loosely speaking, T is disciplined by average tax-net-of-transfer rates and τ by the marginal tax rates at the top.

The flexible functional form with transfers modeled separately from progressive taxes allows to capture two key developments of the U.S. $t\&T$ system over the last decades. First, marginal tax rates have become less progressive, reflected in a lower progressivity parameter in 2010 than in 1950 (Ferriere and Navarro 2025). Second, transfers have risen significantly over this time period, such that average $t\&T$ rates have become more progressive (Heathcote et al. 2020; Splinter 2020).

Transfers data. We measure transfers as reported in NIPA, averaged over four years. We choose 1955-1958 and 2004-2007 to avoid bracketing the Korean War and the Great Recession. Our measure of transfers include the following NIPA categories: food stamps; Supplemental Security Income (SSI); refundable tax credits; unemployment insurance, workers' compensation and temporary disability insurance; family assistance; general assistance; energy assistance; other assistance; and Medicaid.

Spending data. We measure spending as reported in NIPA, averaged over the same years as for the transfers. The measure includes federal, state, and local spending, net of transfers.

C.2 Calibration: Untargeted Moments

Table C.1 reports a subset of untargeted aggregate moments. Table C.2 reports the distributions of wealth and income, both in the data and in the model, in 1950 and 2010.

Table C.2: Income and Wealth Distributions

1950	**Income** Share by Quintile				
Model	6%	10%	14%	21%	50%
Data (SCF+)	6%	11%	15%	21%	48%
2010	**Income** Share by Quintile				
Model	4%	8%	12%	19%	56%
Data (SCF+)	4%	9%	13%	21%	53%
1950	**Wealth** Share by Quintile				
Model	0%	2%	7%	17%	74%
Data (SCF+)	0%	1%	4%	11%	84%
2010	**Wealth** Share by Quintile				
Model	0%	1%	5%	14%	80%
Data (SCF+)	-1%	1%	3%	10%	87%

Notes: Table C.2 compares income and wealth shares by quintile of the respective distribution in model and data. Data comes from the SCF+.

C.3 Risk Aversion, Wealth Effects and MPCs

C.3.1 Relationship between Risk Aversion, Wealth Effects and MPCs

First-order intratemporal condition in the household's optimization problem (6) gives:

$$v'(n) - u_e(e)\theta(1-\mathcal{T}'(\theta n)) = 0, \quad \text{with } v'(n) = Bn^{\varphi}.$$

We implicitly differentiate this equation to obtain

$$\begin{aligned}
v''(n)\frac{\partial n}{\partial T} - u_{ee}(e)\frac{\partial e}{\partial T}\theta(1-\mathcal{T}'(\theta n)) + u_e(e)\theta^2\mathcal{T}''(\theta n)\frac{\partial n}{\partial T} &= 0\\
-\frac{v''(n)}{\theta}\eta - u_{ee}(e)\theta(1-\mathcal{T}'(\theta n)) \times \text{MPC} - u_e(e)\theta\mathcal{T}''(\theta n)\eta &= 0\\
-\eta\left(\frac{1}{\theta}v''(n) + u_e(e)\theta\mathcal{T}''(\theta n)\right)e - u_{ee}(e)\frac{v'(n)}{u_e}e \times \text{MPC} &= 0\\
-\eta\left(\frac{1}{\theta}\frac{v''(n)}{v'(n)} + \frac{\theta u_e(e)}{v'(n)}\mathcal{T}''(\theta n)\right)e + \text{RRA} \times \text{MPC} &= 0
\end{aligned}$$

which delivers equation (8).

C.3.2 Dynamic Model: Measurement of Wealth Effects and MPCs

Wealth effects. We compare model-implied wealth effects on household earnings with evidence provided by Golosov et al. (2024). They merge data from lottery winnings with earnings data covering the universe of U.S. taxpayers. Our preferred measure of comparison is the average reduction in per-adult total labor earnings in the five years following a lottery win. They report a reduction of labor earnings by \$2.3 per \$100 of lottery wealth.

In the model, we expose households to a wealth shock corresponding to the average post-tax win size reported by Golosov et al. (2024). This win size is $\$181,200$ in 2016 dollars. Then, we simulate two panels of households, one of which is exposed to this wealth shock and one of which is not. We compute the average difference between the two groups and obtain a drop of labor earnings of \$2.2 per \$100 of additional wealth.

MPCs. The empirical literature typically computes MPCs as the consumption response to a small windfall gain. To be comparable with that approach, we expose households in the model calibrated to the year 2010 to a one-time wealth shock of \$500. We compute MPCs as the differences in the expenditure after the wealth shock relative to a counterfactual in which no such shock occurs. We report the population average.

C.4 Mirrlees Parameterizations

Table C.3 summarizes the calibrated parameters of the Mirrlees setup we present in Section 4.4. Preference parameters $\{\varepsilon_j; \sigma\}$ rely on the micro estimates from Comin et al. (2021), while the parameters $\{\Omega_j\}$ are set to match aggregate sector shares. We also keep the other parameters of the utility function, γ and φ, as in the dynamic model. Prices are set to replicate aggregate growth and changes in relative prices over time. We set government parameters to match transfer-to-output ratios, spending-to-output ratios, and the difference in AMTRs between the top-10% and bottom-90% of the distribution. Expenditure distributions by quintiles are comparable across the static and the dynamic models, as reported in Table C.4.

Computation. We compute the optimal $t\&T$ system by iterating on the tax formula (rather than by mechanism design) using a log-spaced grid of 1,000 points (Heathcote and Tsujiyama 2025).

C.5 Pareto Weights

Dynamic Model. The inverse optimum weights that make the 1950 $t\&T$ system optimal are: $\mu = -16.46$ and $\nu = 16.63$. Though there is no guarantee to match exactly the calibrated system, we come very close to matching the observed tax system, with an optimal

Table C.3: Mirrlees Parameterization

Parameter		Baseline	Higher RA	Fiscal Pressure	IA Preferences
Preferences					
γ	Curvature utility	0.77	1.50	0.77	-
$1/\varphi$	Frisch elasticity	0.50	0.50	0.50	0.50
B	Labor disutility	13.00	13.00	13.00	28.00
σ	NH CES parameter	0.30	0.30	0.30	-
ε_A	NH CES parameter	0.10	0.10	0.10	-
ε_G	NH CES parameter	1.00	1.00	1.00	-
ε_S	NH CES parameter	1.80	1.80	1.80	-
Ω_A	NH CES parameter	0.08	0.11	0.08	-
Ω_G	NH CES parameter	1.00	1.00	1.00	-
Ω_S	NH CES parameter	2.42	2.25	2.42	-
Prices					
p_A^{1950}	Price agriculture 1950	1.00	1.00	1.00	1.00
p_G^{1950}	Price goods 1950	1.00	1.00	1.00	1.00
p_S^{1950}	Price services 1950	1.00	1.00	1.00	1.00
p_A^{2010}	Price agriculture 2010	0.28	0.23	0.27	0.21
p_G^{2010}	Price goods 2010	0.15	0.12	0.14	0.11
p_S^{2010}	Price services 2010	0.46	0.39	0.45	0.36
Inequality					
α^{1950}	Pareto tail 1950	4.40	4.40	4.40	4.40
α^{2010}	Pareto tail 2010	3.30	3.30	3.30	3.30
σ_a^{1950}	EMG parameter 1950	0.37	0.48	0.37	0.37
σ_a^{2010}	EMG parameter 2010	0.47	0.61	0.49	0.47
Government					
λ^{1950}	Tax level 1950	0.32	0.28	0.32	0.33
τ^{1950}	Tax progressivity 1950	0.15	0.15	0.15	0.15
T^{1950}	Transfer 1950	0.00	0.01	0.00	0.00
G^{1950}	Spending 1950	0.07	0.10	0.07	0.07
λ^{2010}	Tax level 2010	0.31	0.30	0.33	0.31
τ^{2010}	Tax progressivity 2010	0.09	0.09	0.09	0.09
T^{2010}	Transfer 2010	0.01	0.01	0.01	0.01
G^{2010}	Spending 2010	0.07	0.08	0.07	0.07

Notes: Table C.3 summarizes the calibrated parameters of the Mirrlees setup, for four cases: the benchmark calibration; the robustness calibration with higher risk aversion; the robustness calibration with higher fiscal pressure; and the robustness calibration for IA preferences. Preference parameters for the IA preference case are reported Footnote 25.

Table C.4: Expenditure Distribution in the Dynamic and the Static Model

1950	**Expenditure** Share by Quintile				
Dynamic model	8%	14%	18%	22%	38%
Static model	9%	13%	17%	23%	38%
2010	**Expenditure** Share by Quintile				
Dynamic model	7%	12%	16%	21%	44%
Static model	8%	12%	16%	22%	42%

Notes: Table C.4 compares the expenditure distributions in the static and the dynamic model.

progressivity of 0.11 and a transfer-to-output ratio of 1.3%, similar to their data counterpart.

Static Model. The inverse optimum weights can be derived as a function of types without imposing a functional form in 1950, as described in Lockwood and Weinzierl (2016). As in the dynamic model, we then compute Pareto weights in 2010 as a function of the position in the distribution, keeping weights identical at the first and last grid points—for which the measure is close to zero.

C.6 Robustness: Dynamic Model

When expressing the Pareto weights in terms of productivity, the inverse optimum weights that make the 1950 $t\&T$ system optimal are: $\mu = -29.22$ and $\nu = 29.46$.[27] The optimal $t\&T$ system generates an optimal progressivity of 0.11 and a transfer-to-output ratio of 1.2%, similar to their data counterpart. We use the productivity Pareto weights to compute the optimal $t\&T$ system in two cases: first, with only rising inequality; and second, when also accounting for rising living standards.

Results resemble those obtained under the expenditure Pareto weights. Redistribution $\mathcal{R}$ increases to 44 p.p. with inequality only, and to only 29 p.p. when also accounting for rising living standards: rising living standards dampen by 75% the increase in redistribution. The transfer-to-output ratio increases to 3.8% with rising inequality only, and to 2.1% when also accounting for rising living standards.

It is reassuring to find similar results when assuming Pareto weights function of productivities, which are exogenous. Yet, expenditure-based Pareto weights have two advantages: the expenditure distribution is much smoother, and expenditure is a better proxy for overall inequality.

[27] We follow Tauchen (1986) to discretize the productivity process in 18 points.